Quantum Computing via Local Control

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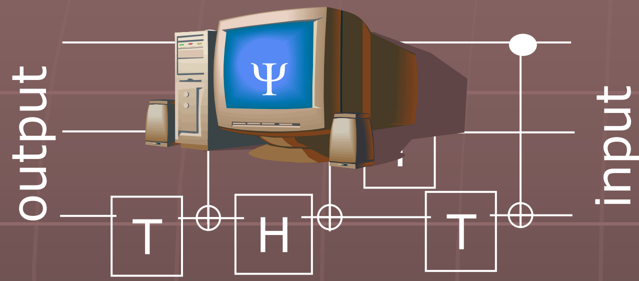
Quantum Circuits = Unitary Transformations



$\longleftrightarrow$ output $\begin{pmatrix} \psi_o \end{pmatrix} = \begin{pmatrix} U \end{pmatrix} \begin{pmatrix} \psi_i \end{pmatrix}$ input

- "Rule" (logical operation) U(t) same for all input
- The Schrödinger Equation: $$\dot{\psi} = -\frac{i}{\hbar} H \psi$$
- We can formally Solve: $$\psi(t) = T e^{-\frac{i}{\hbar}\int_0^t H dt} \psi(0)$$
- Unitary propagator U(t) creates mapping between ψ(0) and ψ(t): $$\psi(t) = U(t)\ \psi(0)$$

The Unitary Control Problem

External laser Field *E(t)*

$$\psi(t) = Te^{-\frac{i}{\hbar}\int_0^t H(E(t'))dt'}\,\psi(0)$$

$$= U[E]\;\psi(0)$$

- $U(t)$ is determined by the laser field $E(\cdot)$:

$$U(t) = U([E],t)$$

- Given a desired $U(T)=O$ can we find a field $E(\cdot)$ that produces it?
- Inverse problem ←→ Control problem

[1] C.M. Tesch and R. de Vivie-Riedle, PRL 89, 157901 (2002)
[2] J.P. Palao and R. Kosloff, PRL 89, 188301 (2002)

Control of a State vs. Control of a Transformation

- What is usually done in quantum control:

- Control of a State:

find $E(t)$ such that

$\Psi_f \leftarrow \Psi_i$.

$$\dot{\psi} = -\frac{i}{\hbar} H[E]\psi$$

Controls the evolution of *one* state

- What we have here – a harder problem !

- Control of a Transformation:

find $E(t)$ such that

$\Psi_f^{(1)} = U\,\Psi_i^{(1)}$,

$\Psi_f^{(2)} = U\,\Psi_i^{(2)}$,

$\vdots \qquad \vdots$

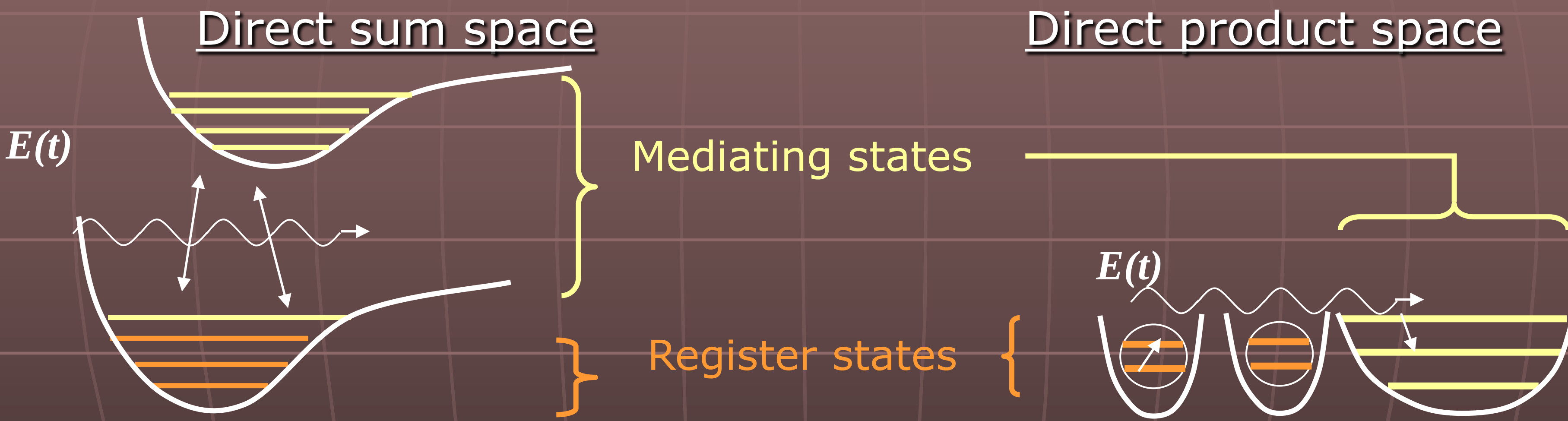
$\Psi_f^{(n)} = U\,\Psi_i^{(n)}$.

$$\dot{U} = -\frac{i}{\hbar} H[E]U$$

Controls simultaneously the evolution of *all* possible states and phases

Quantum Register and Mediating States

- System=Register+Mediating states
- Two alternative realizations:



- Objective: Produce Target Unitary Transformation on register *without* intermediate population of auxiliary mediating states

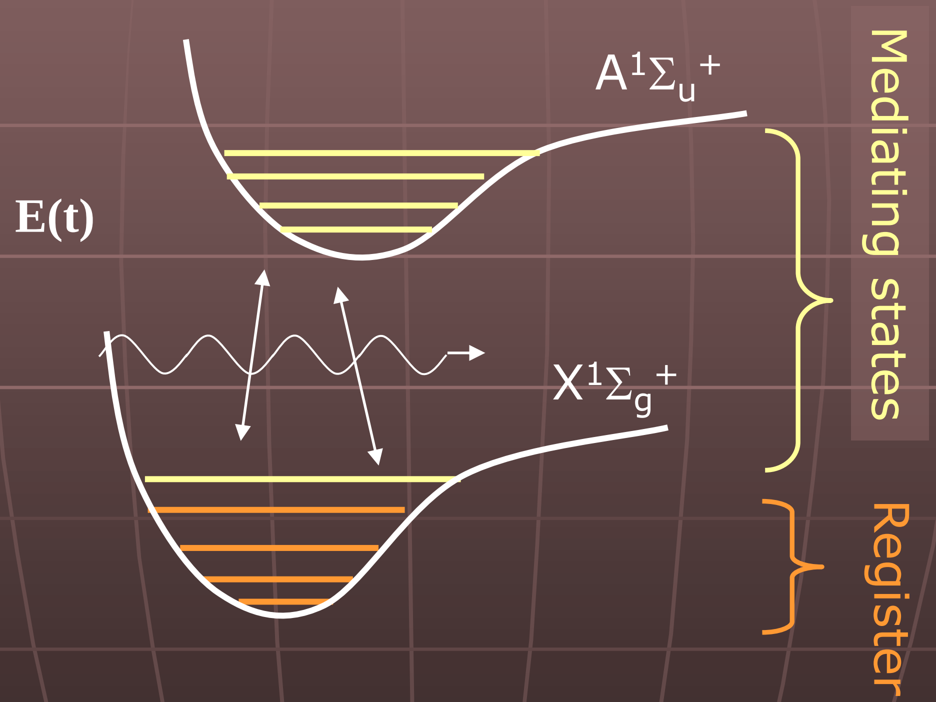
Projection onto Register

- Separable Unitary transformation on space:

Entire Hilbert Space $\left\{ \right.$ $$U = \begin{pmatrix} U_R & \\ & U_M \end{pmatrix}$$ $\left.\begin{matrix} \\ \\ \end{matrix}\right\}$ Register states, $\}$ Mediating states

- Define P a projection operator onto the quantum register sub-manifold: $U_R = PUP$

The Model: Producing Unitary Transformations on the Vibrational Ground Electronic States of Na_2



$$H = H_0 + H_{int} \quad , \quad H_{int} = \begin{pmatrix} & \mu E \\ \mu E^* & \end{pmatrix}$$

Definition of Constrained Unitary Control Problem

- System equation of motion:

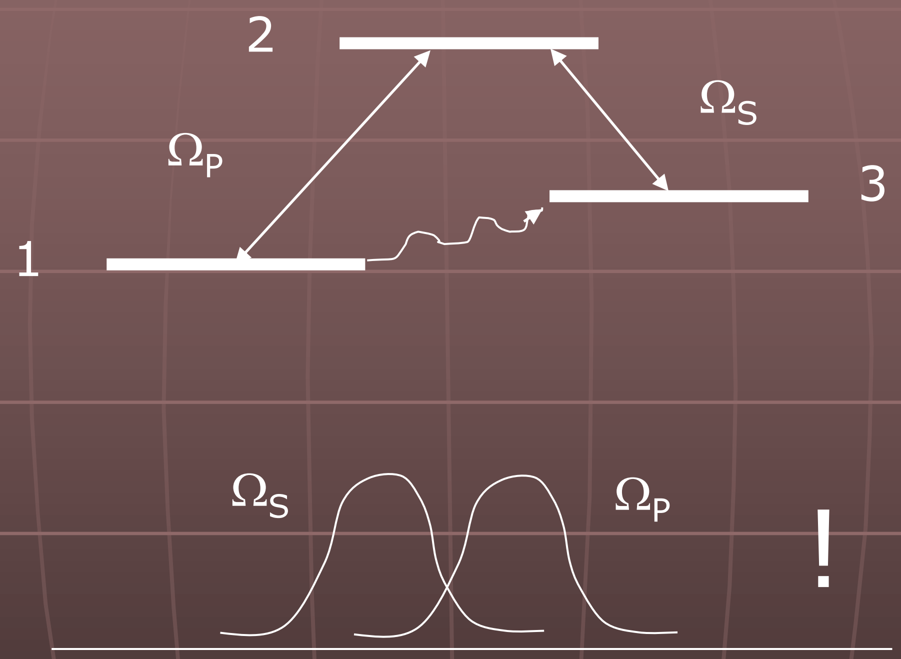
$$\dot{U} = \frac{1}{i\hbar} HU \qquad U(0) = I$$

- Control: laser field $E(t)$
- Objective: target unitary transformation O_R

  Maximize $J = |Tr(O_R{}^{\dagger} U_R(T))|^2$
- Constraint: No depopulation of register

  Conserve $C = Tr(U_R{}^{\dagger} U_R)$

Motivation:
Stimulated Raman Adiabatic Passage (STIRAP)



Bergmann et al. (1990).
Bergmann, Theuer and Shore, Rev Mod. Phys. 70, 1003 (1998).
V. Malinovsky and D. J. Tannor, Phys. Rev. A 56, 4929 (1997).

Local Optimization Method

At each point in time:

- Enforce constraint C

  $dC/dt = \mathrm{Imag}(g\,E(t)) = 0$

  → $E(t) = a\,g^*$

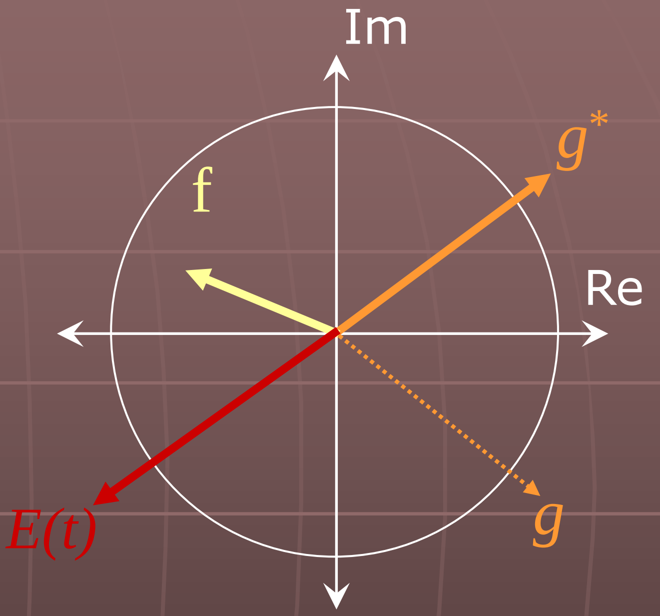
  → direction

- Monotonic increase in Objective J

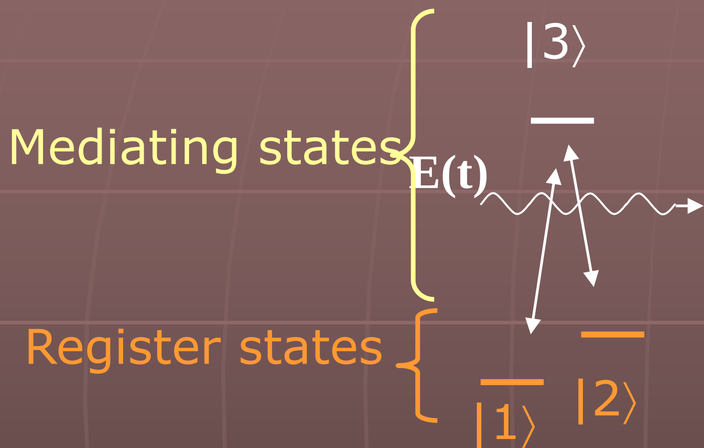
  $dJ/dt = \mathrm{Real}(f\,E(t)) = a\,\mathrm{Real}(f\,g^*) > 0$

  → $a = \mathrm{Real}(f\,g^*)$

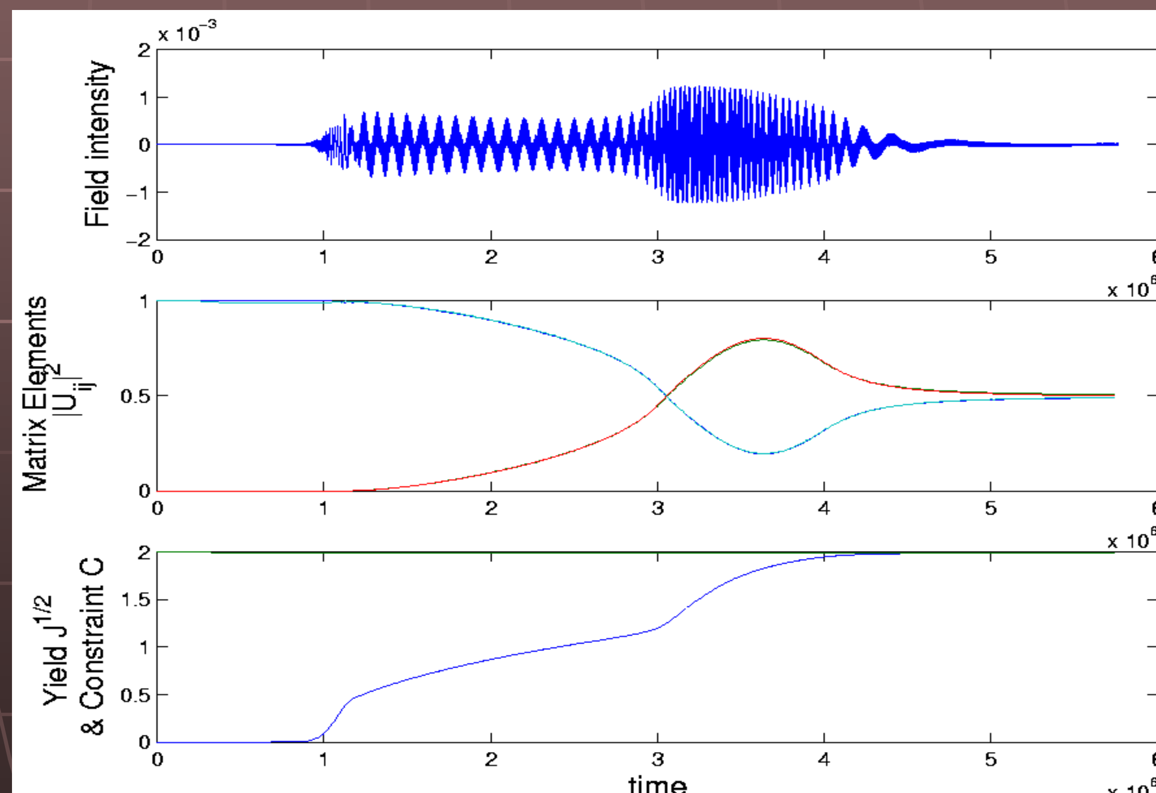
  → Sign and magnitude



Creating a Hadamard Gate in a Three-Level Λ-System



$$H = \frac{1}{\sqrt{2}}\begin{pmatrix} 1 & 1 \\ 1 & -1 \end{pmatrix}$$



Femto-second pulse shaping

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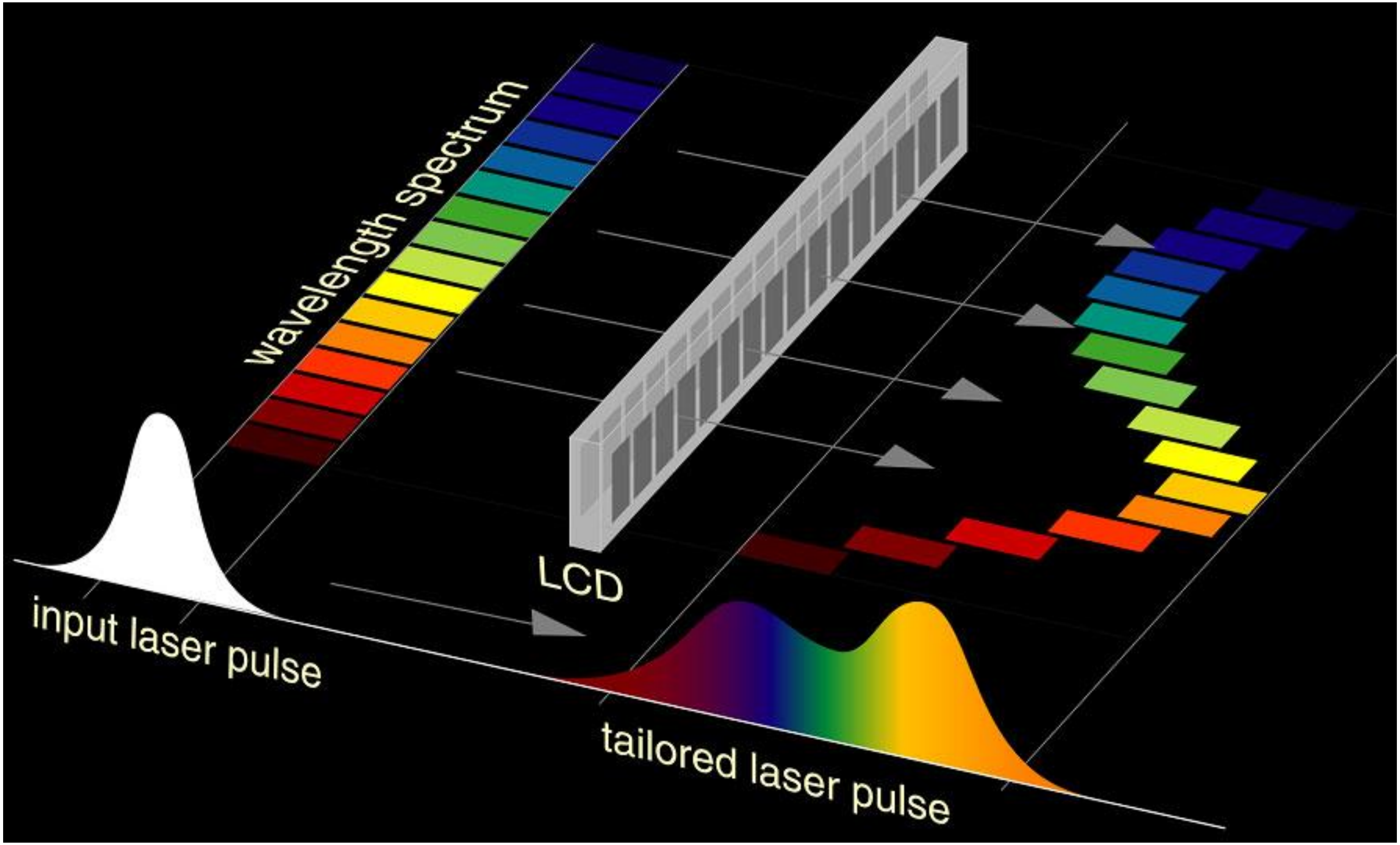
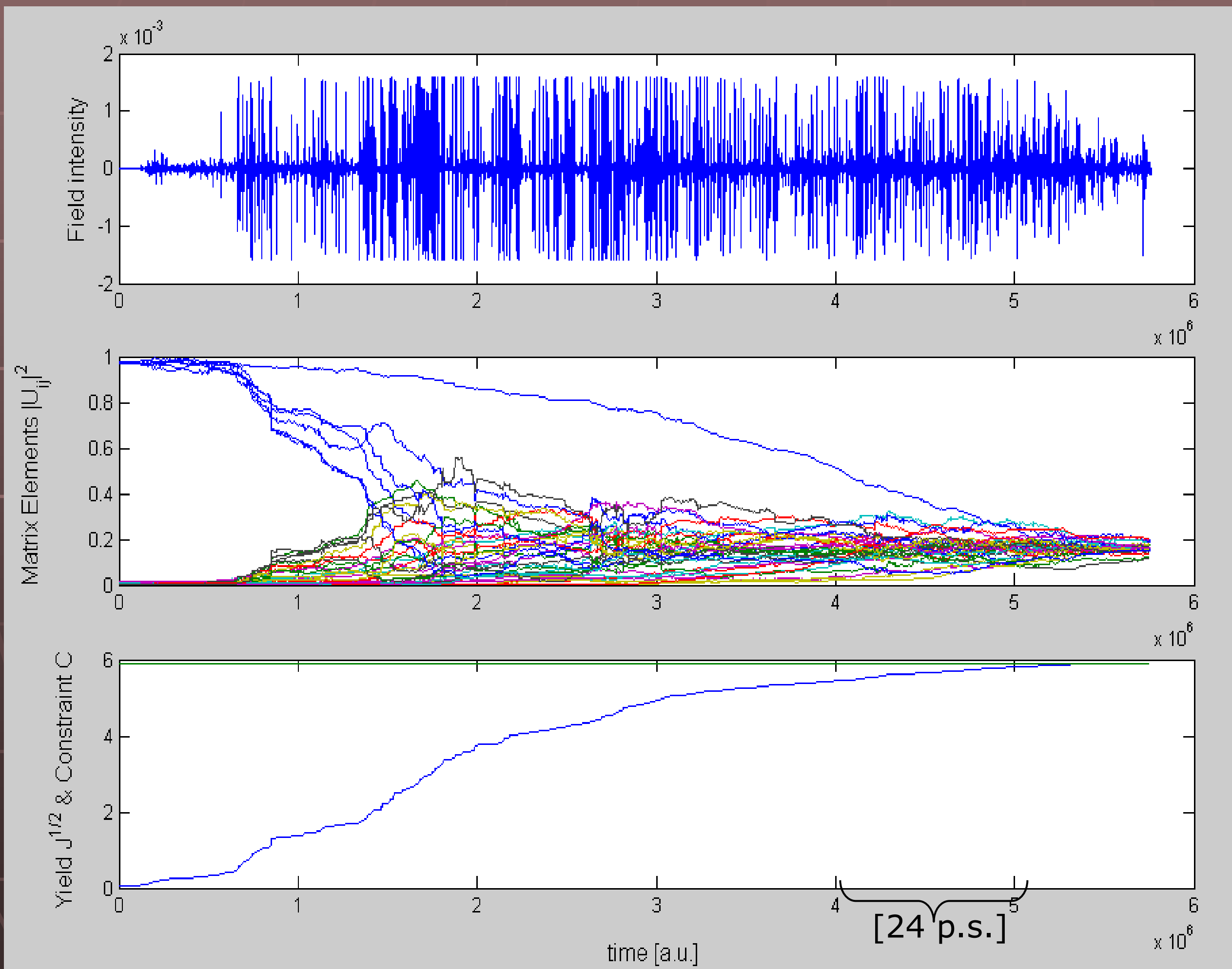


Fourier Transform on a Quantum Register: with (7+3) level sub-manifold of Na_2;



$$w = e^{2\pi i/6}$$

$$FT = \frac{1}{\sqrt{6}} \begin{pmatrix} w & w^2 & w^3 & w^4 & w^5 & 1 \\ w^2 & w^4 & 1 & w^2 & w^4 & 1 \\ w^3 & 1 & w^3 & 1 & w^3 & 1 \\ w^4 & w^2 & 1 & w^4 & w^2 & 1 \\ w^5 & w^4 & w^3 & w^2 & w & 1 \\ 1 & 1 & 1 & 1 & 1 & 1 \end{pmatrix}$$

Direct-Sum vs. Direct-Product Space (separable transformations)

- Direct Sum

$$U = U_R \oplus U_M$$

$$\begin{pmatrix} U_R & \\ & U_M \end{pmatrix}$$

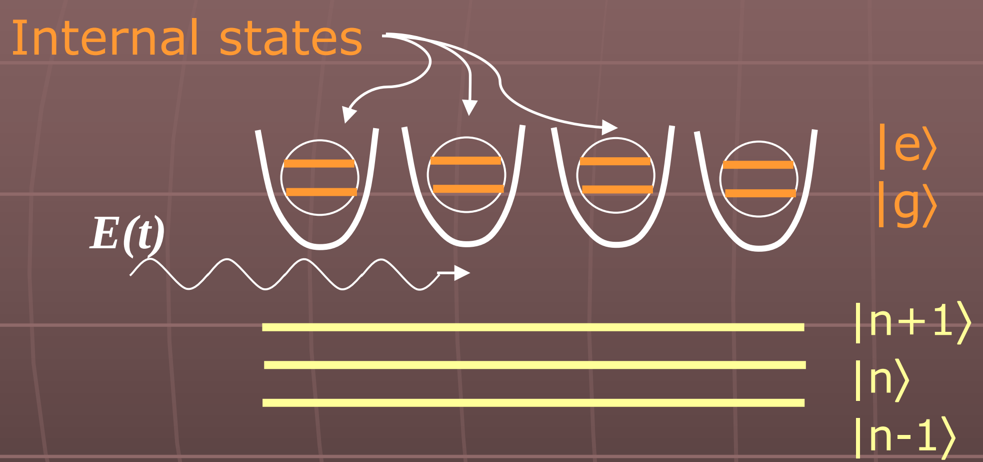
- Direct product

$$U = U_R \otimes U_M$$

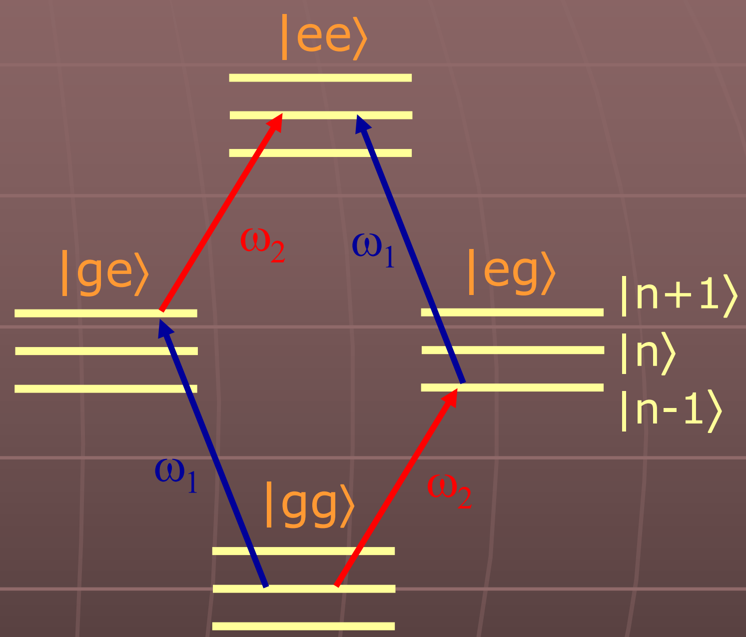
$$\begin{pmatrix} U_{R_{11}} U_M & \cdots & U_{R_{1n}} U_M \\ \vdots & \ddots & \vdots \\ U_{R_{n1}} U_M & \cdots & U_{R_{nn}} U_M \end{pmatrix}$$

Ion-Trap Quantum Gates

Atoms in linear trap



External Center of mass modes



Problem: Entanglement of the Quantum register with the external modes!

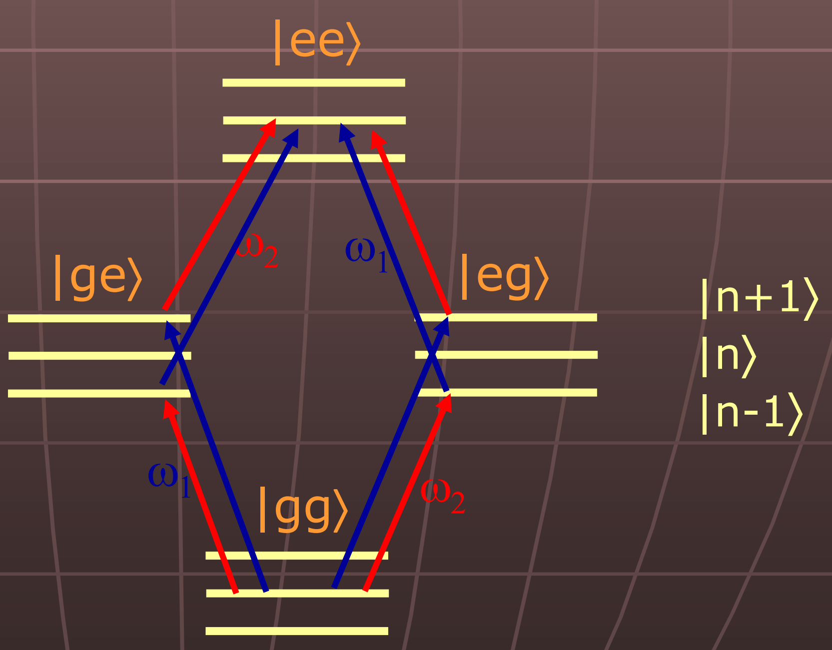
[1] J.I. Cirac and P. Zoller, PRL **74**, 4091 (1995)
[2] A. Sørensen and K. Mølmer, PRL **82**, 1971 (1999)
[3] T. Calarco, U. Dorner, P.S. Julienne, C.J. Williams and P. Zoller, PRA **70**, 012306, (2004)

Liouville-Space Formulation

- Projection $\mathcal{P}$ onto register must trace out the environment producing, in general, mixed states on the register.
- Liouville space description is required!
  - Space: $\mathcal{H} \longrightarrow \mathcal{L},$
  - Density Matrix: $\rho \longrightarrow |\rho\rangle\rangle = |\rho_R\rangle \otimes |\rho_E\rangle,$
  - Inner product: $\mathrm{Tr}(\rho^\dagger \sigma) \longrightarrow \langle\langle \rho | \sigma \rangle\rangle$
  - Super Operators: $[H,\rho] \longrightarrow \mathcal{H}|\rho\rangle\rangle,$ $U\rho U^\dagger \quad \mathcal{U}|\rho\rangle\rangle$
  - Evolution Equation: $\dot{\mathrm{U}} = -\frac{i}{\hbar}\mathrm{HU},$

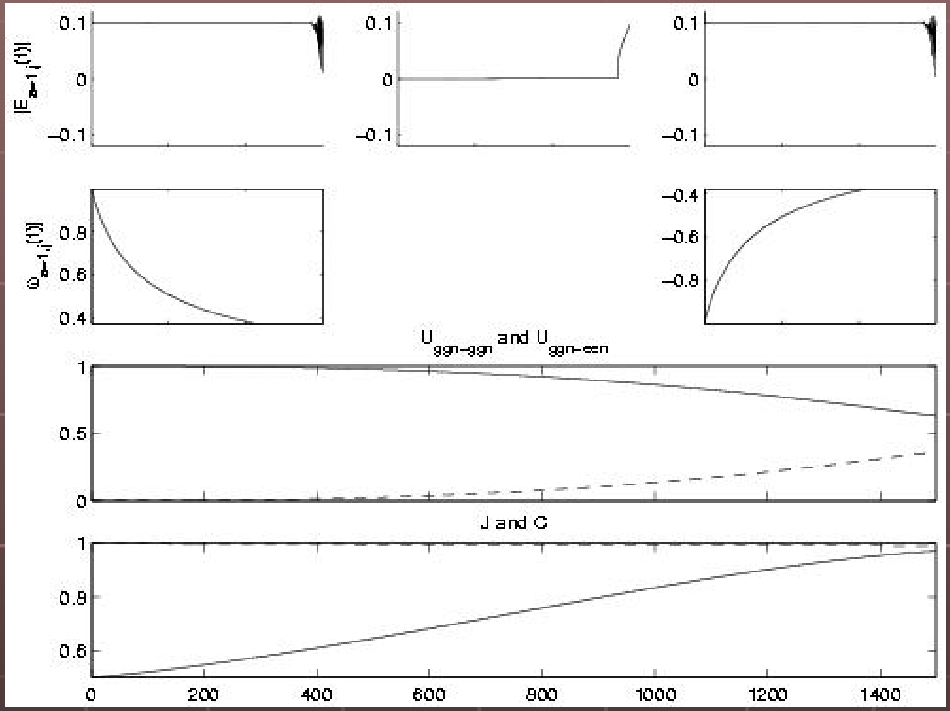
Sørensen-Mølmer Scheme

$$H_{\text{int}} = \underbrace{\sum_{j=1,2} \frac{\hbar\Omega_j}{2} e^{-i\omega_j t}}_{\text{Field}} \underbrace{\sigma_{+j}}_{\text{internal}} \underbrace{e^{i\eta(a+a^+)}}_{\text{external}} + H.C.$$



Local Control (Initial) Results for a two-qubit entangling gate

Fields (amp,phase) and evolution of propagator:



- We assumed each pulse is near-resonant with one of the sidebands
- We fixed the total summed intensity
- Results close to the Sørensen-Mølmer scheme

Summary

- Control of unitary propagators implies *simultaneously* controlling *all* possible states in system
- We devised a Local Control method to eliminate undesired population leakage
- We considered two general state-space structures:
  - •Direct Sum →E.g.:* Hadamard on a Λ system,
    * SU(6)-FT on Na_2
  - •Direct Product→E.g.:* Sørensen-Mølmer Scheme to directly produce arbitrary2-qubit gates