

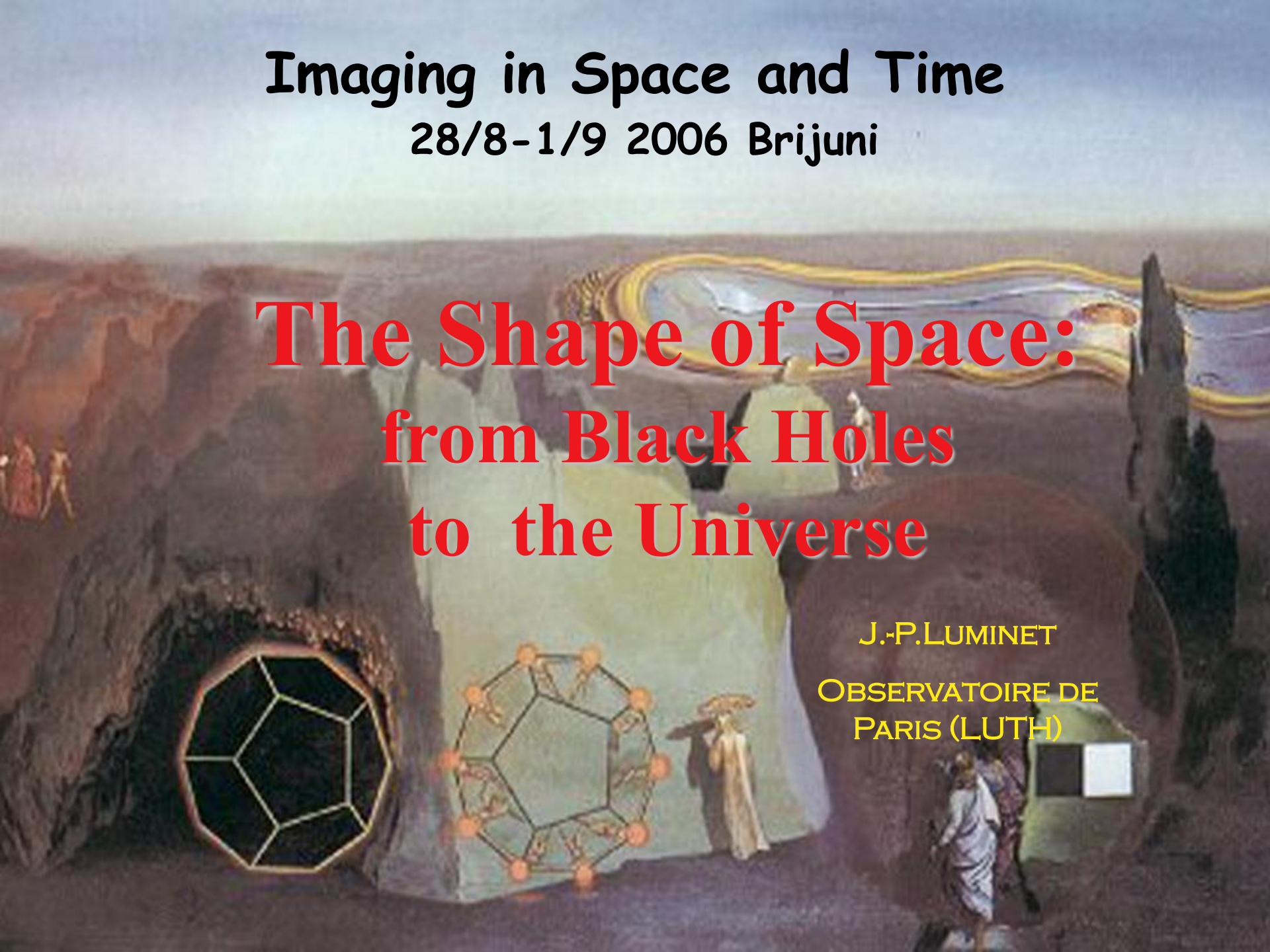
Imaging in Space and Time

28/8-1/9 2006 Brijuni

The Shape of Space: from Black Holes to the Universe

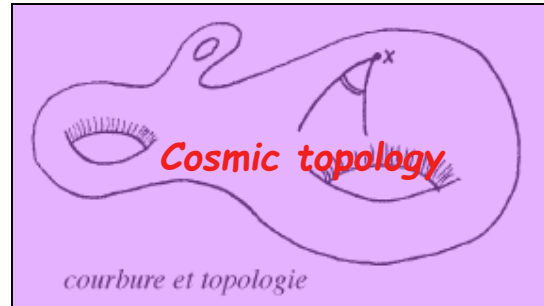
J.-P. LUMINET

OBSERVATOIRE DE
PARIS (LUTH)



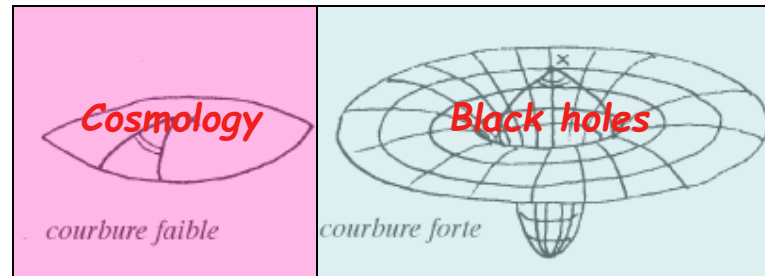
4 levels of geometry

Niveau 1



Topologie cosmique

Niveau 2



Relativité générale,
cosmologie

Niveau 3



Mécanique classique,
Relativité restreinte

Niveau 4



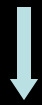
Gravitation quantique,
Théories d'unification



General Relativity

$$G_{ij} = k T_{ij}$$

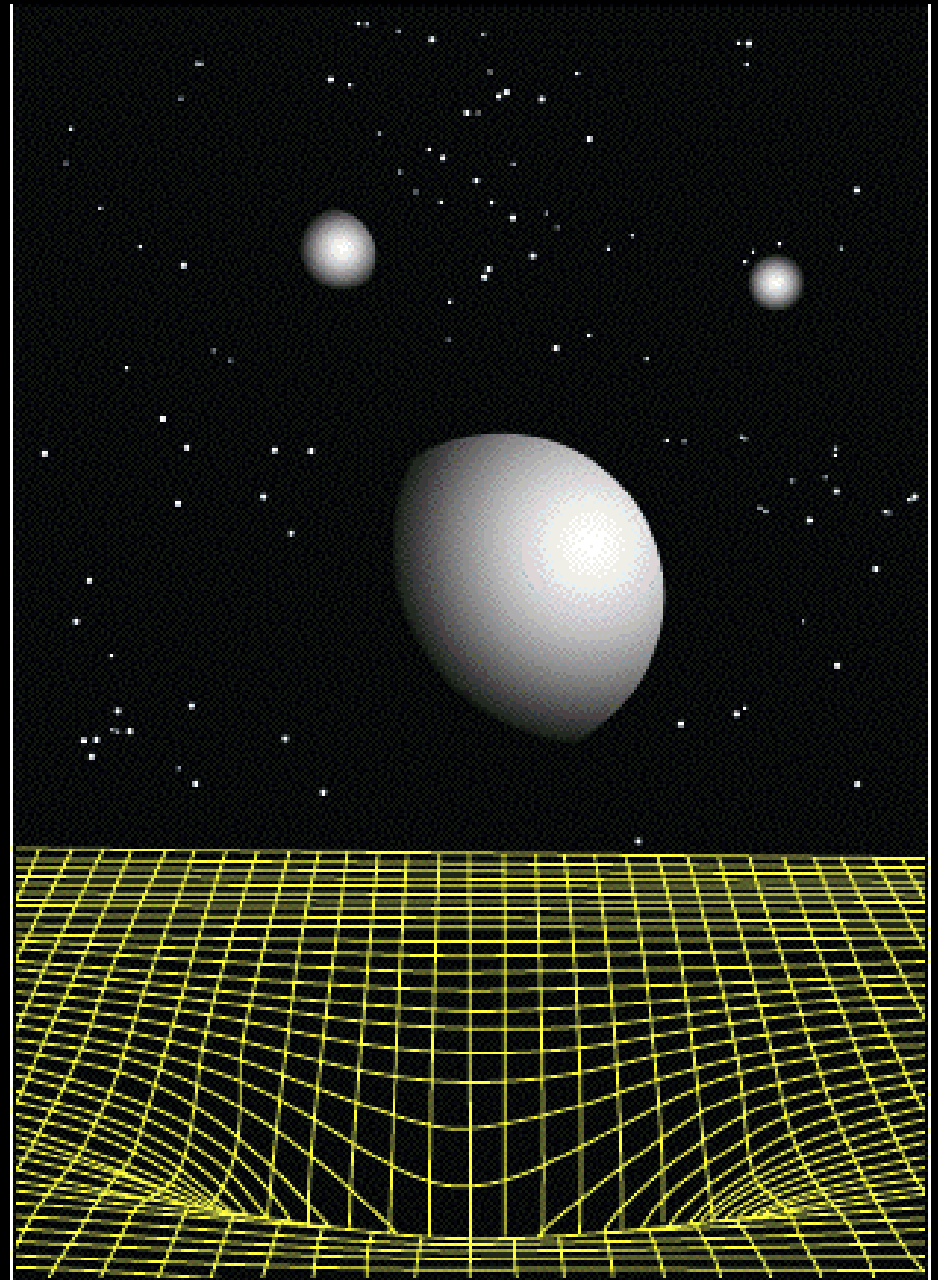
geometry = matter-energy

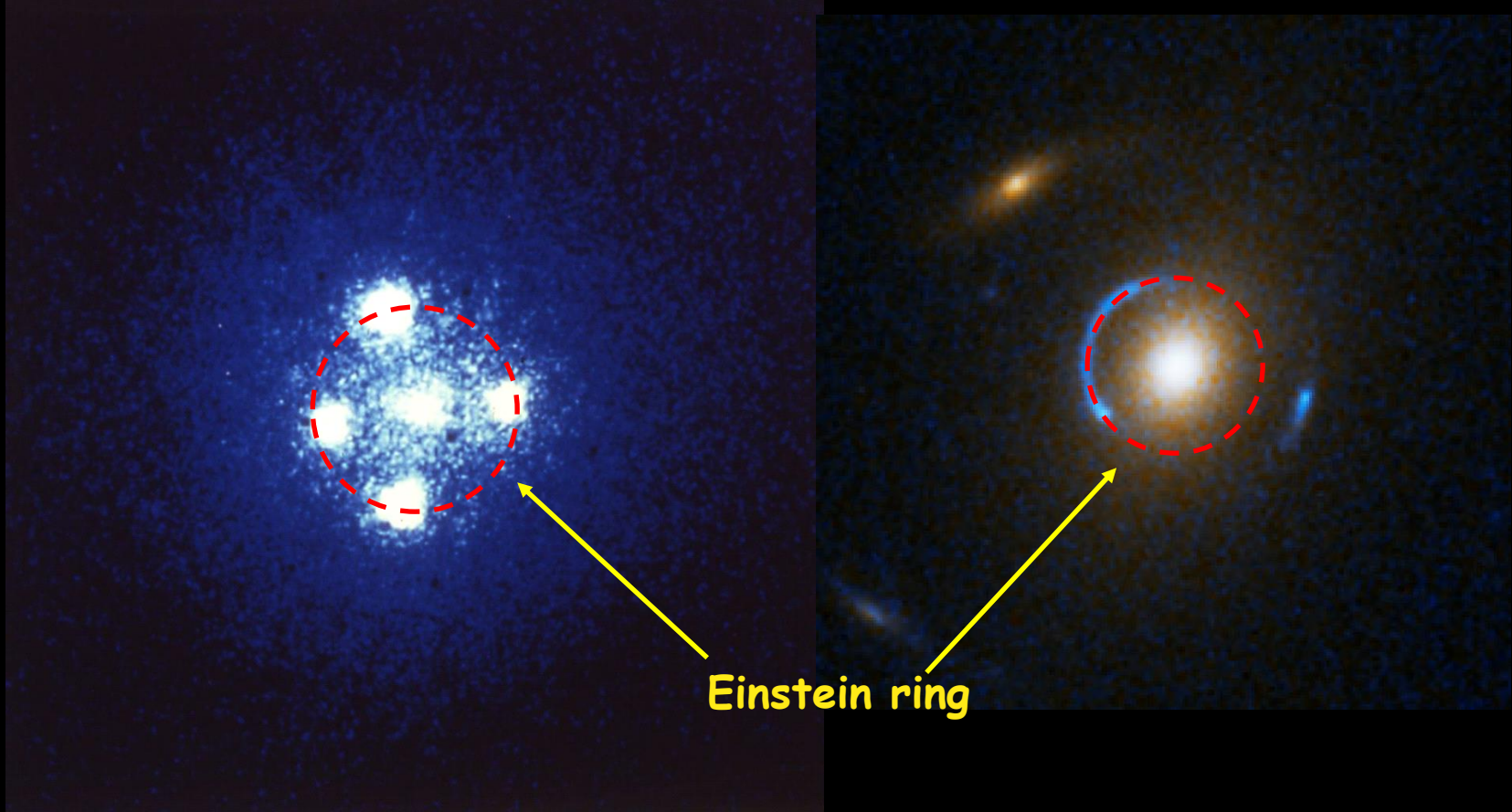


$$ds^2 = g_{ij} dx^i dx^j$$

spacetime metric

gravity =
spacetime curvature





Einstein ring

Gravitational
lensing

If $M_* > 30 M_\odot$

BLACK HOLE !

Imaging Black Holes



Newtonian spacetime



curved spacetime

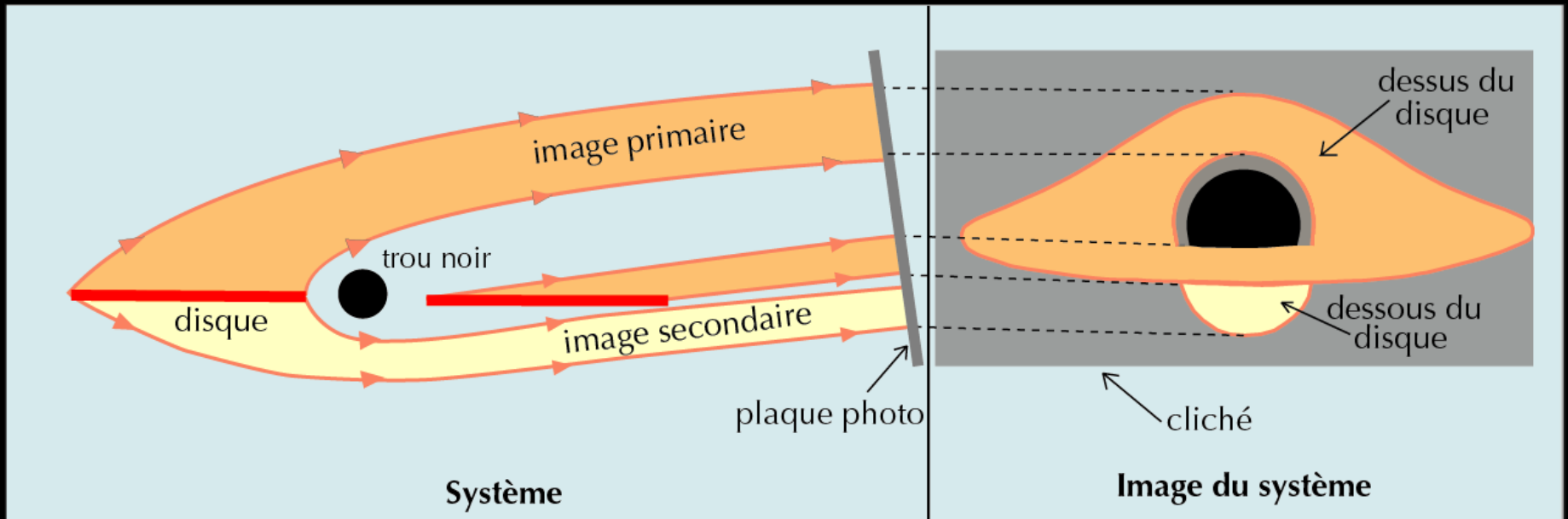
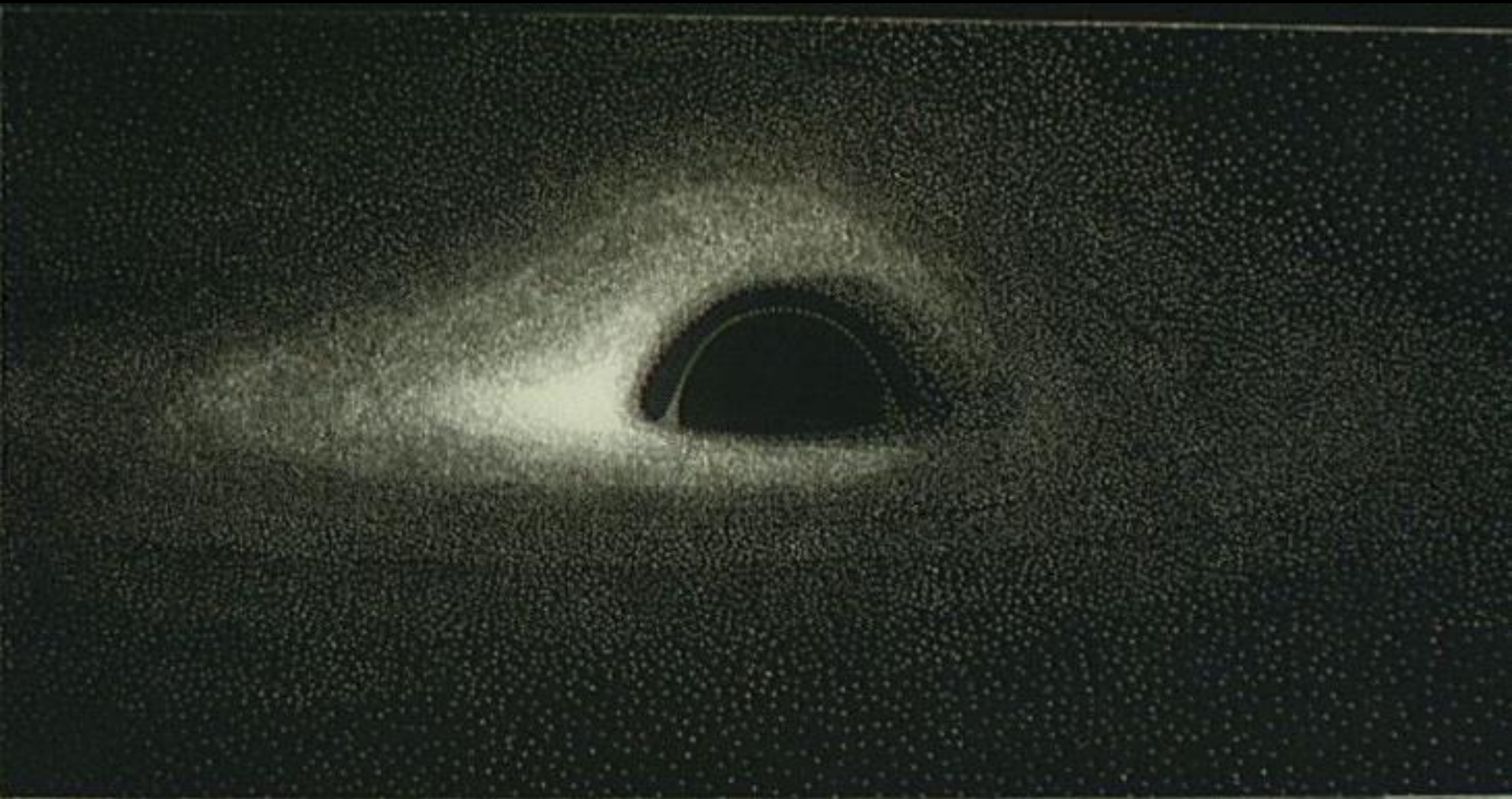


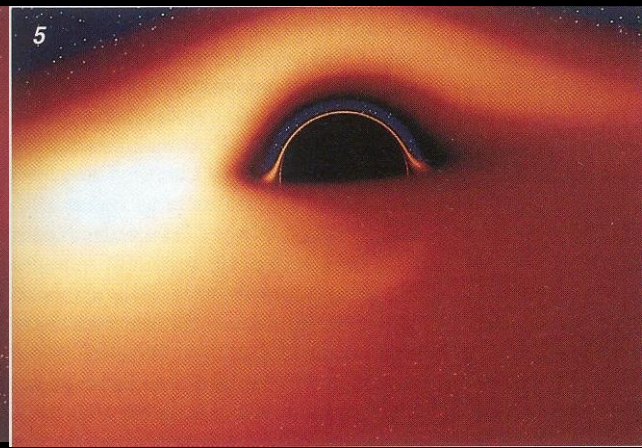
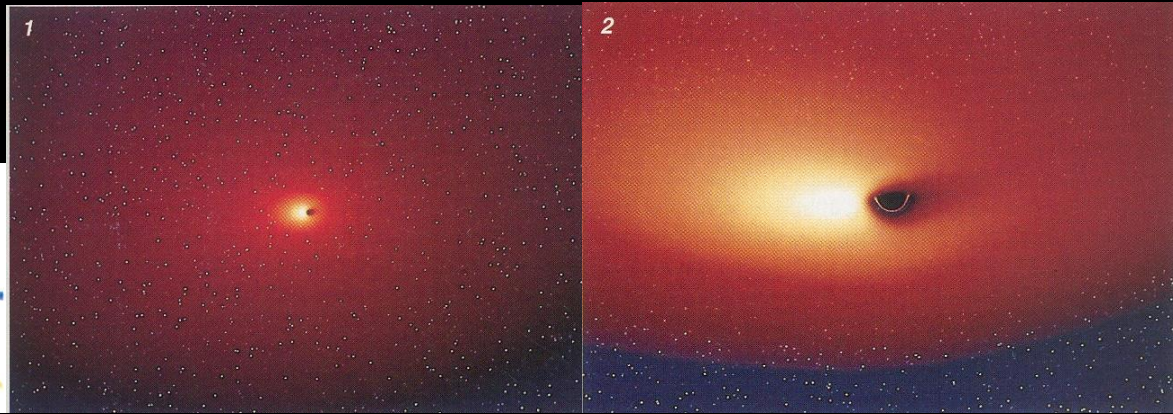
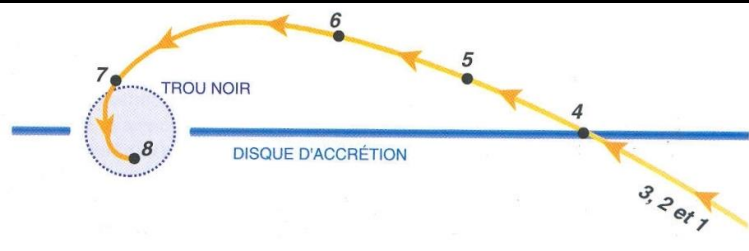
Image of a spherical black hole with thin accretion disk

(J.-P. Luminet, 1979)



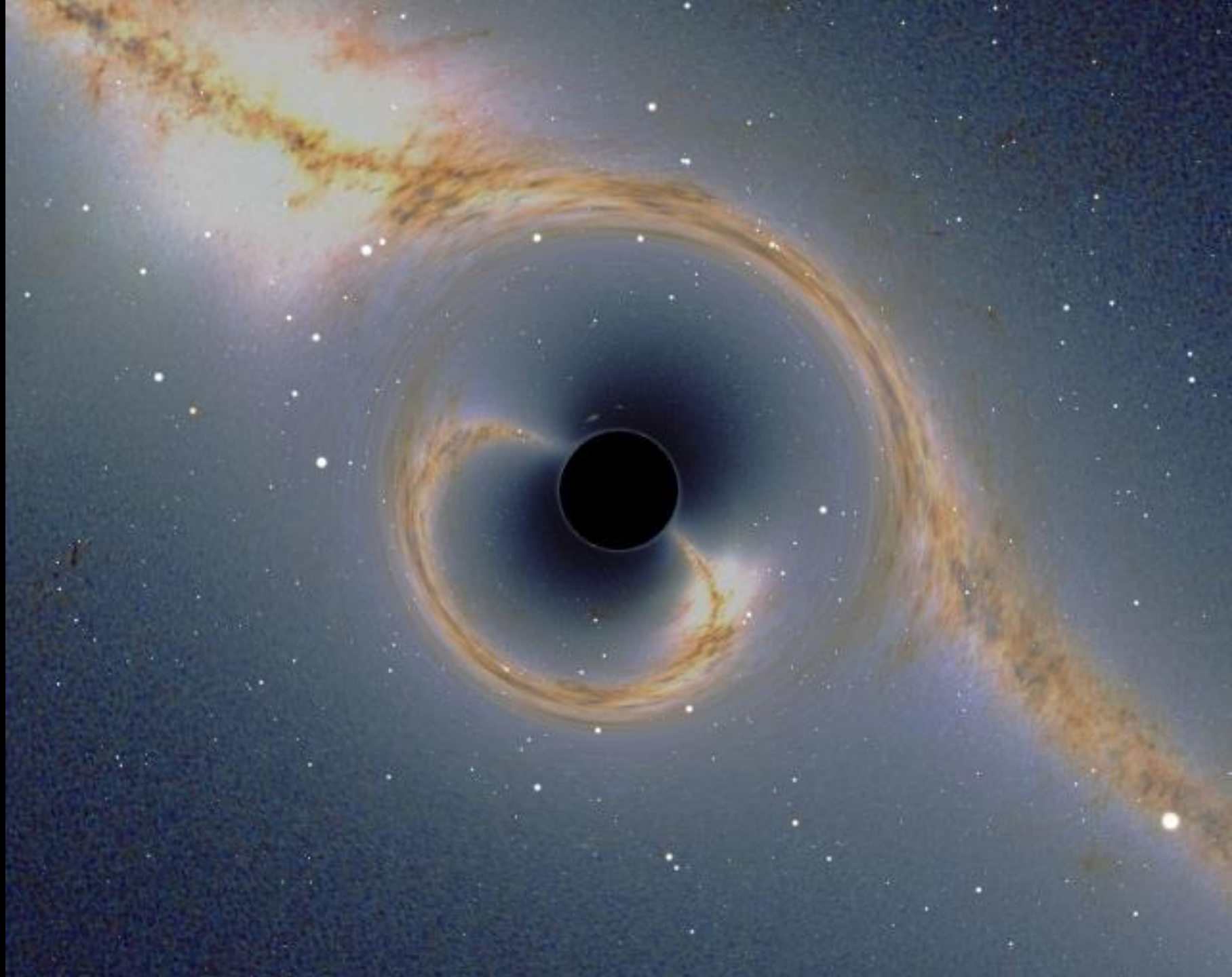
Flight into a black hole

(J.A.Marck, 1993)





Black hole in front of Milky Way
(Riazuelo, 2006)



Castor & Pollux

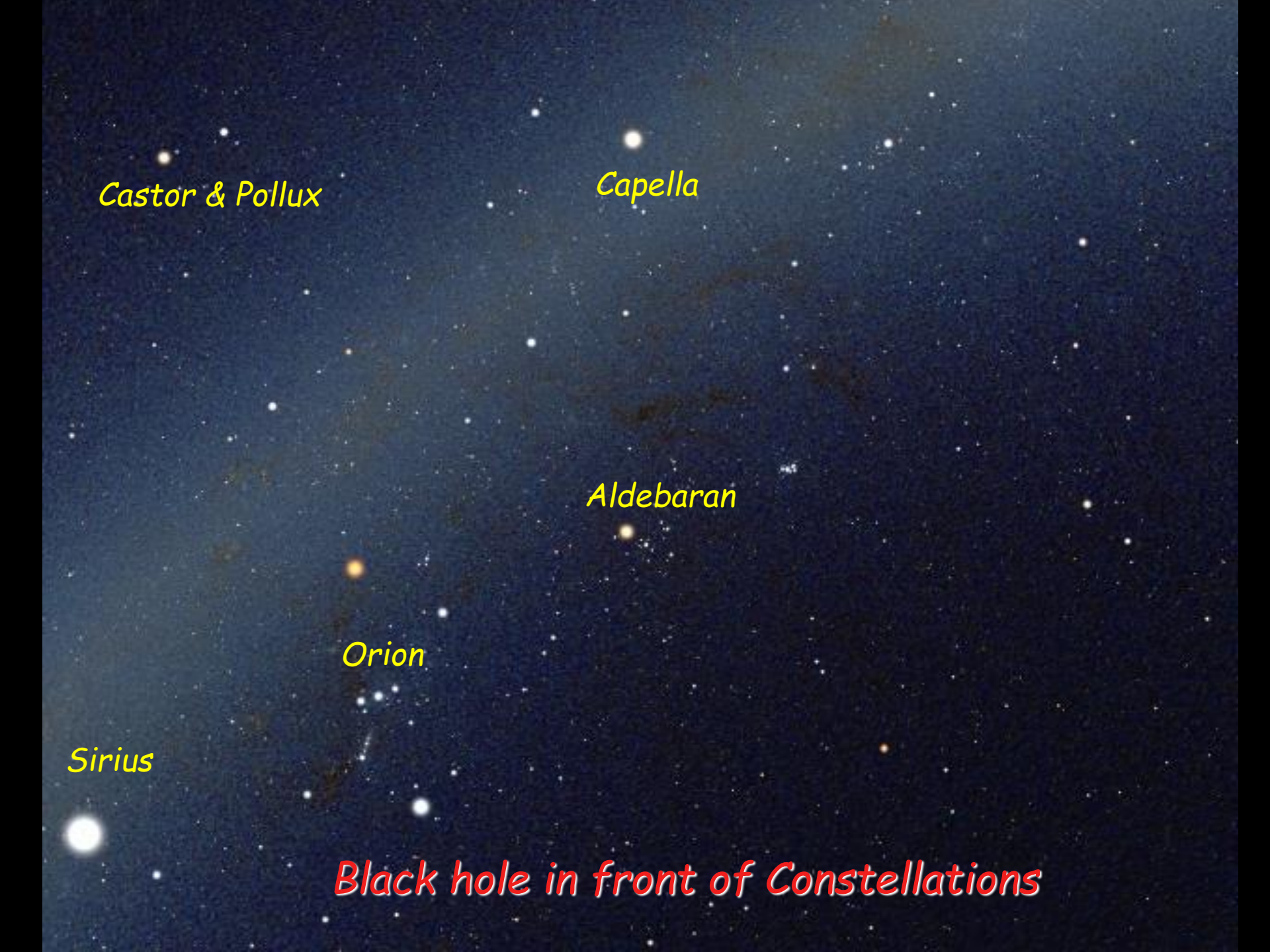
Capella

Aldebaran

Orion

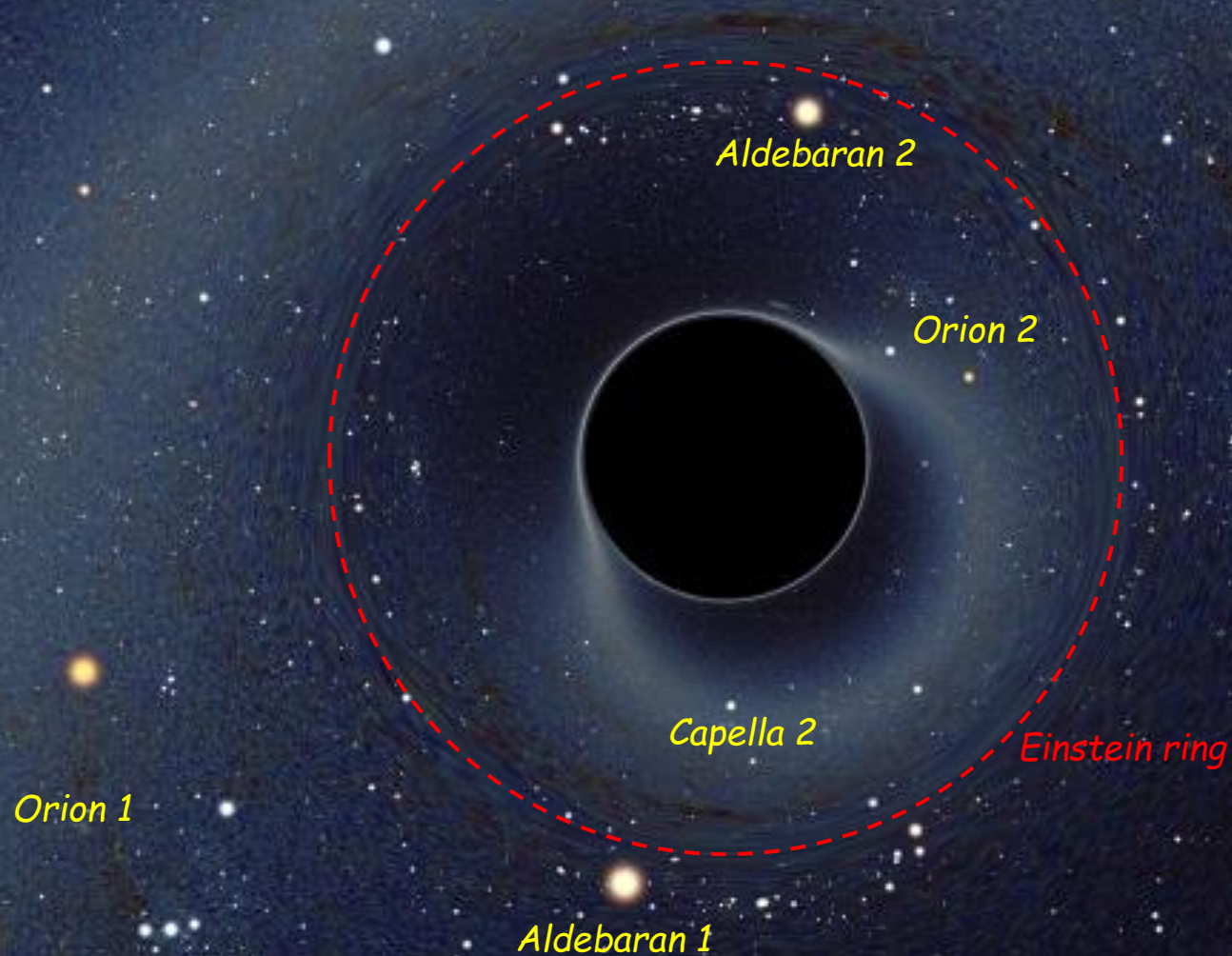
Sirius

Black hole in front of Constellations



Capella 1

Imaging spacetime : light cones



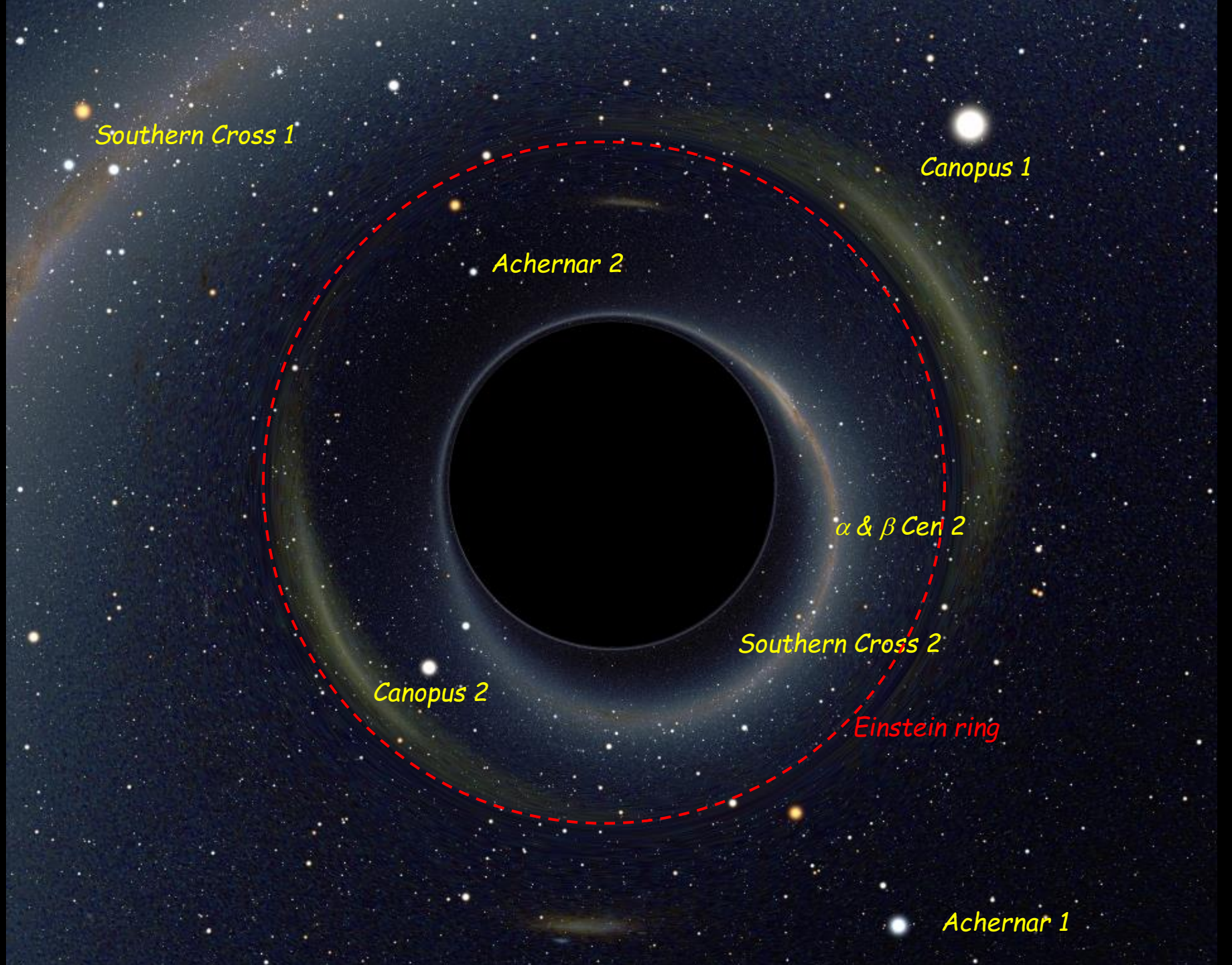
Southern Cross

Canopus

α & β Cen

Achernar

Black hole in front of Magellanic Clouds



Southern Cross 1

Canopus 1

Achernar 2

α & β Cen 2

Southern Cross 2

Canopus 2

Einstein ring

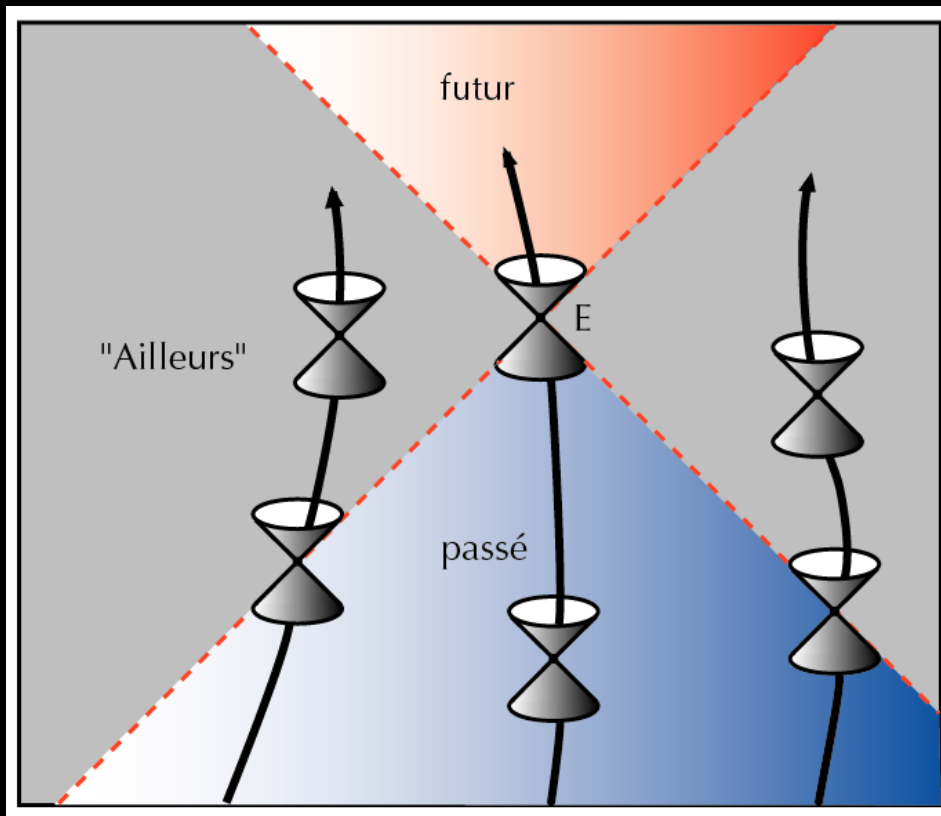
Achernar 1

See movie 1



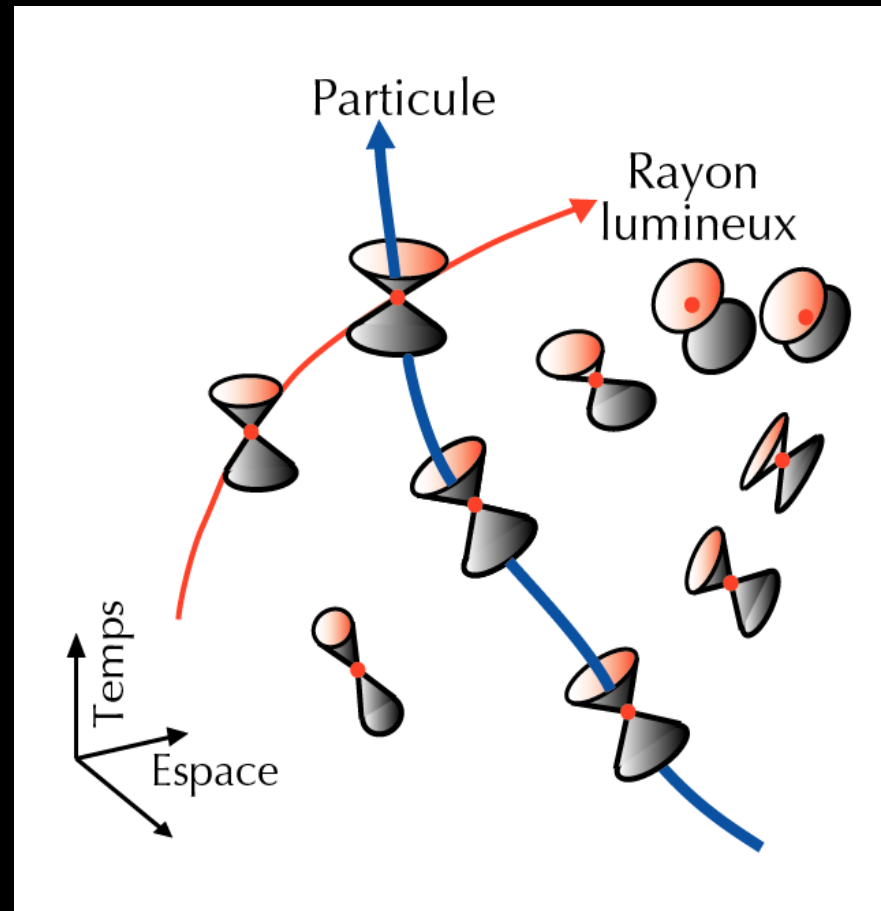
Black hole in front of Magellanic Clouds

Imaging spacetime : light cones



Flat (Minkowski) spacetime

Curved spacetime

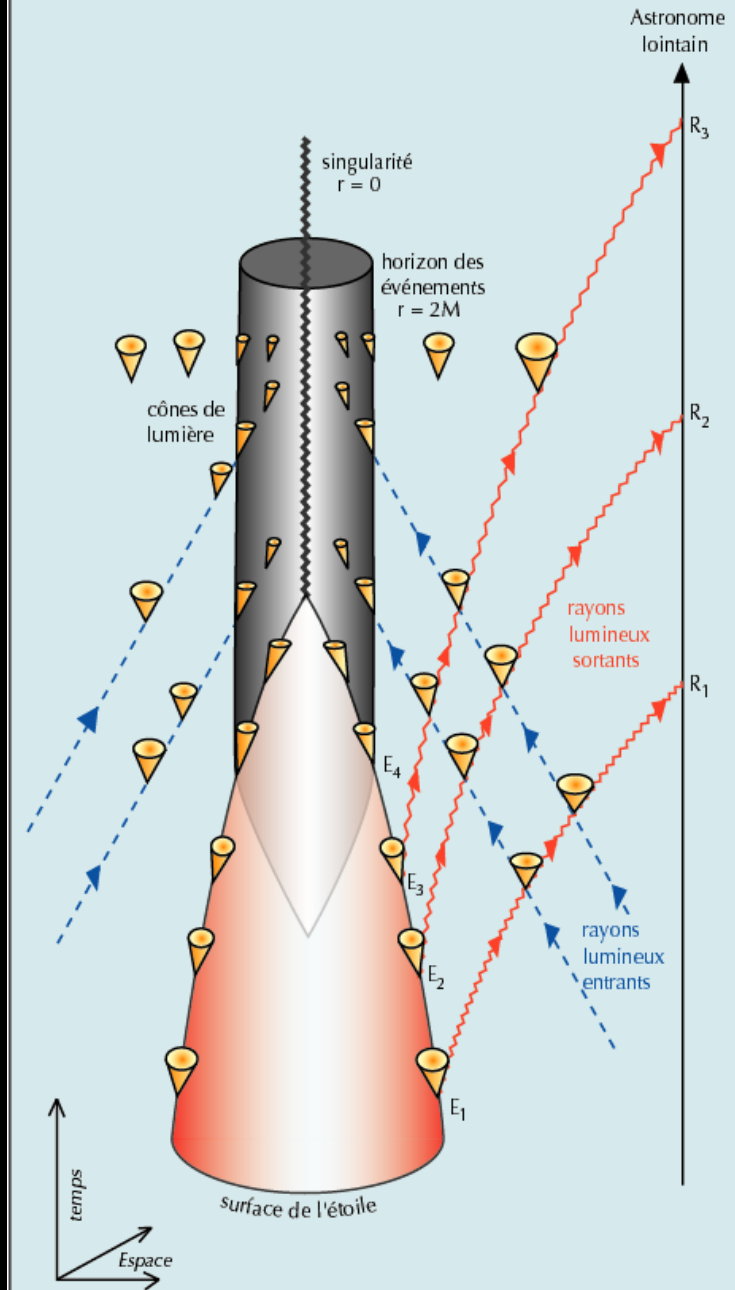


Gravitational collapse to a Schwarzschild black hole

metric:

Schwarzschild radius:

→ Event horizon



Embedding

Step 1: Schwarzschild metric outside mass M ($G=c=1$):



Step 2: Equatorial section
Time section



Curved 2-geometry:

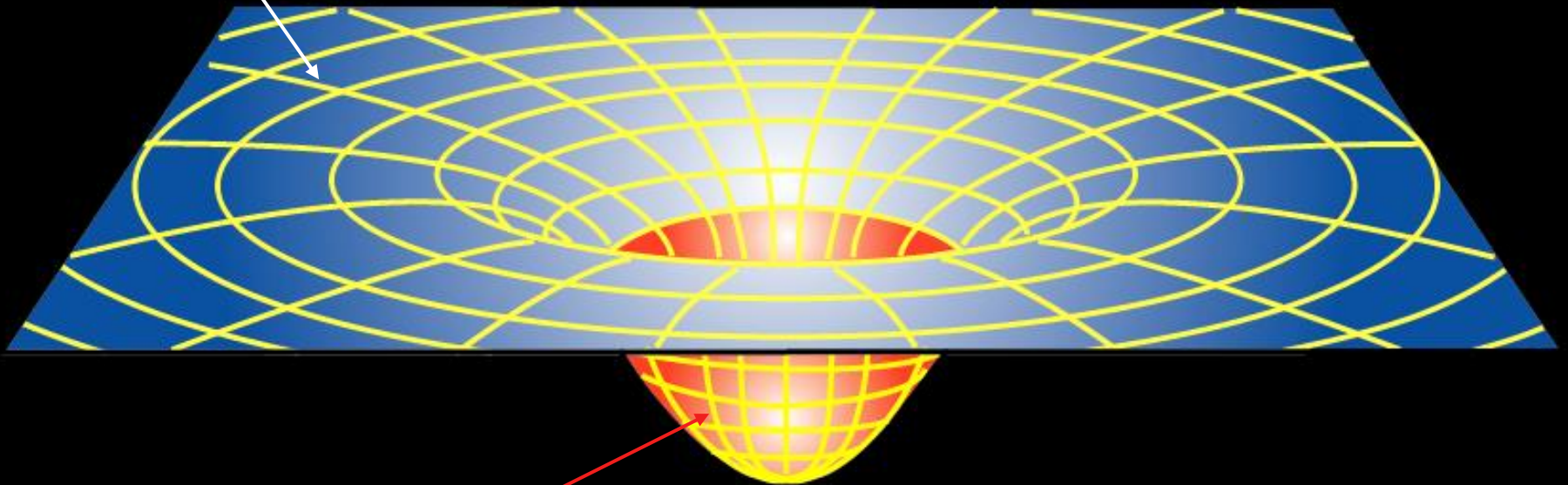


Step 3: Embedding in 3D
Euclidian space



Result for ordinary star ($R_* > 2M$)

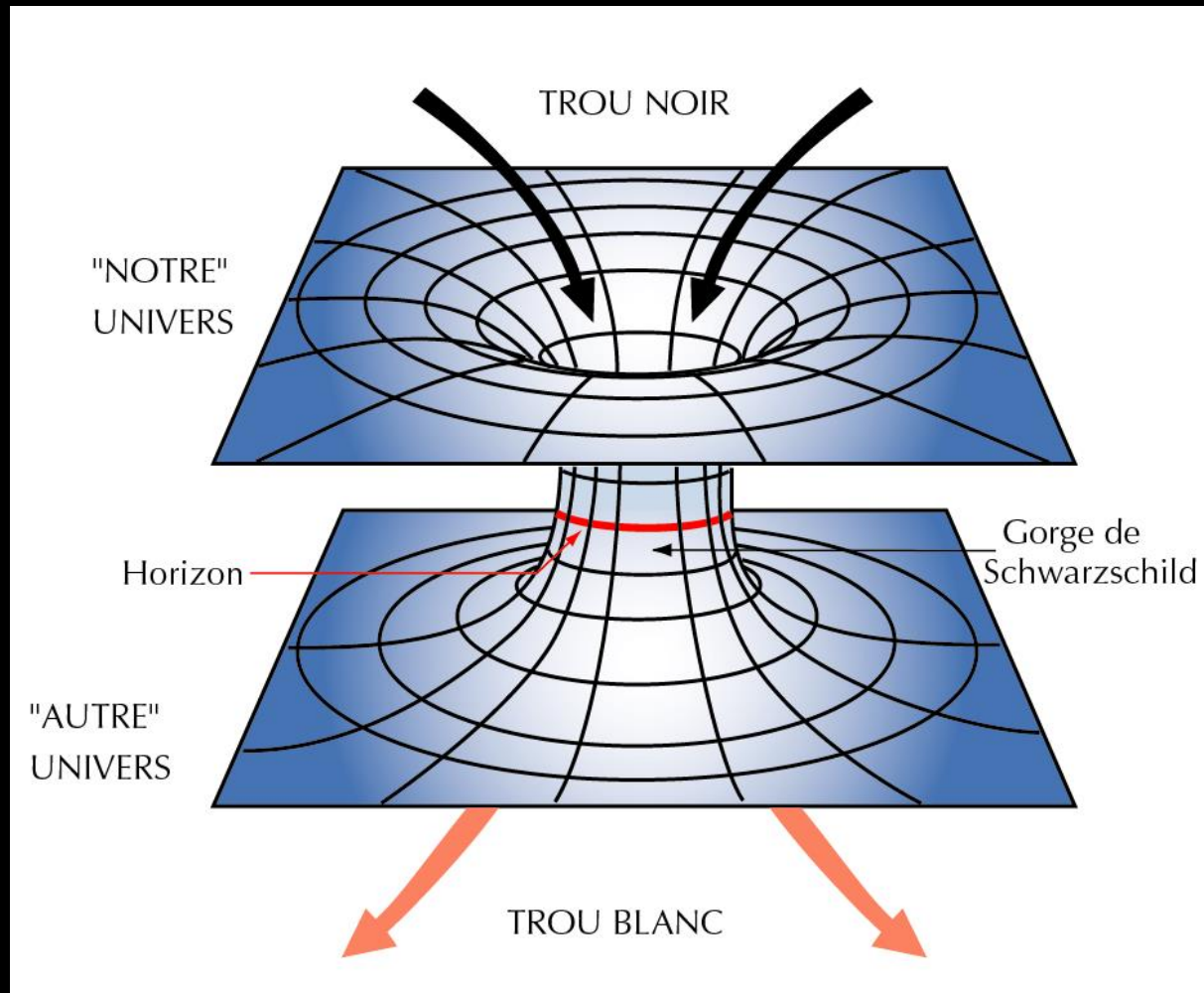
Outer solution (asymptotically flat)



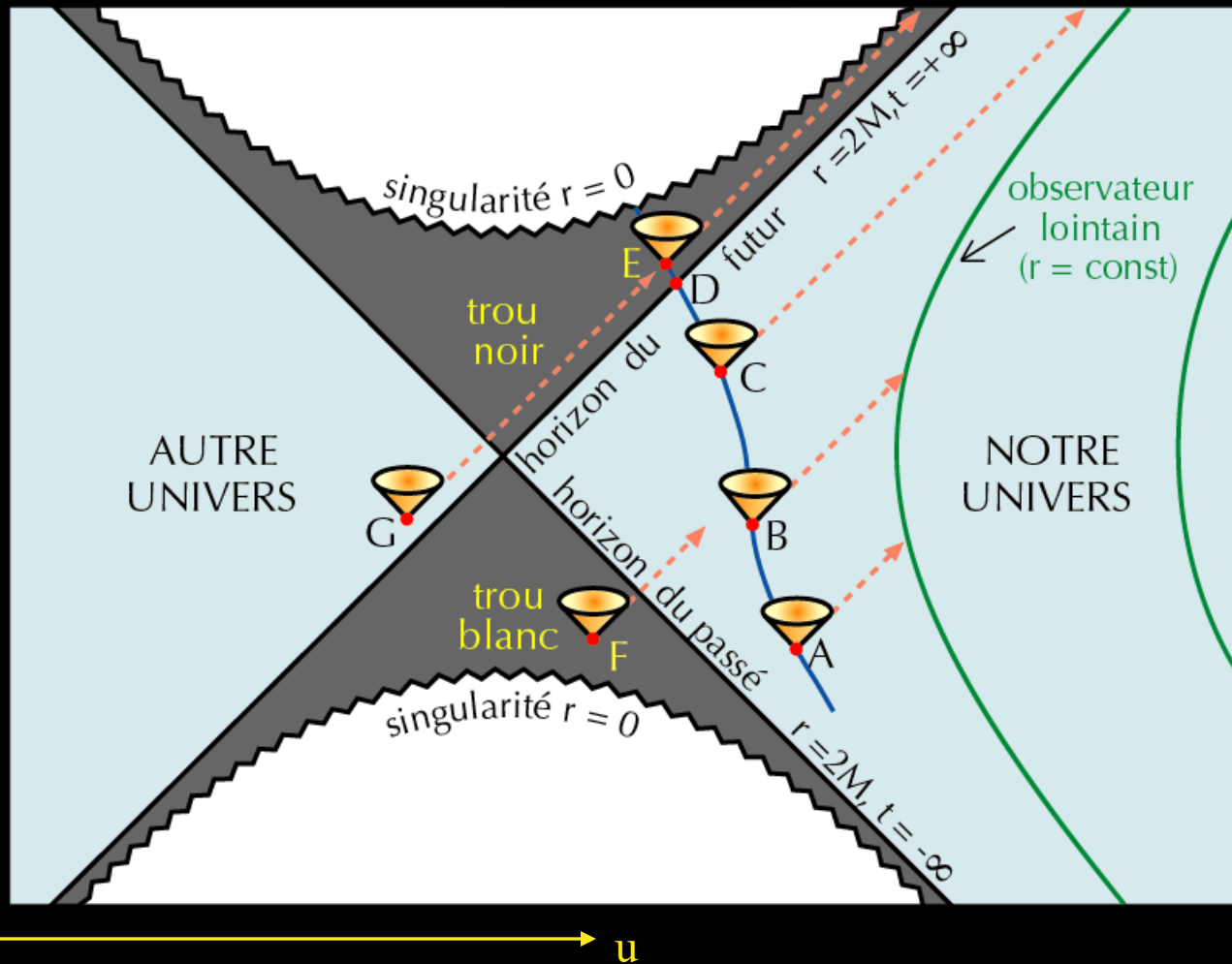
Inner solution (regular)

Result for black hole

Outer solution only
(Flamm paraboloid)



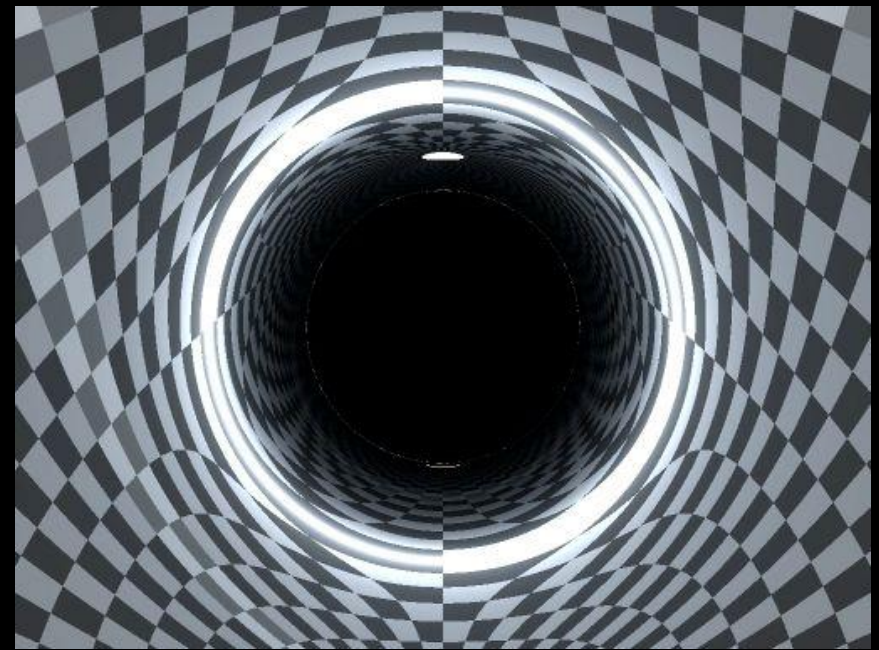
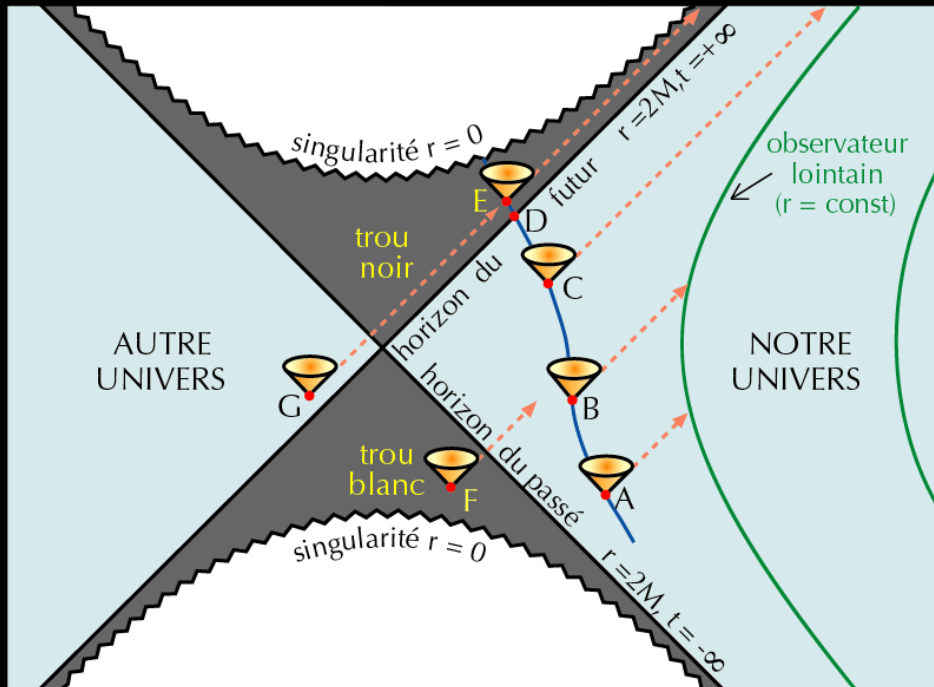
Spherical black hole in Kruskal coordinates



Flight into a static black hole

Radial photons

(A.Riazuelo, 2006)

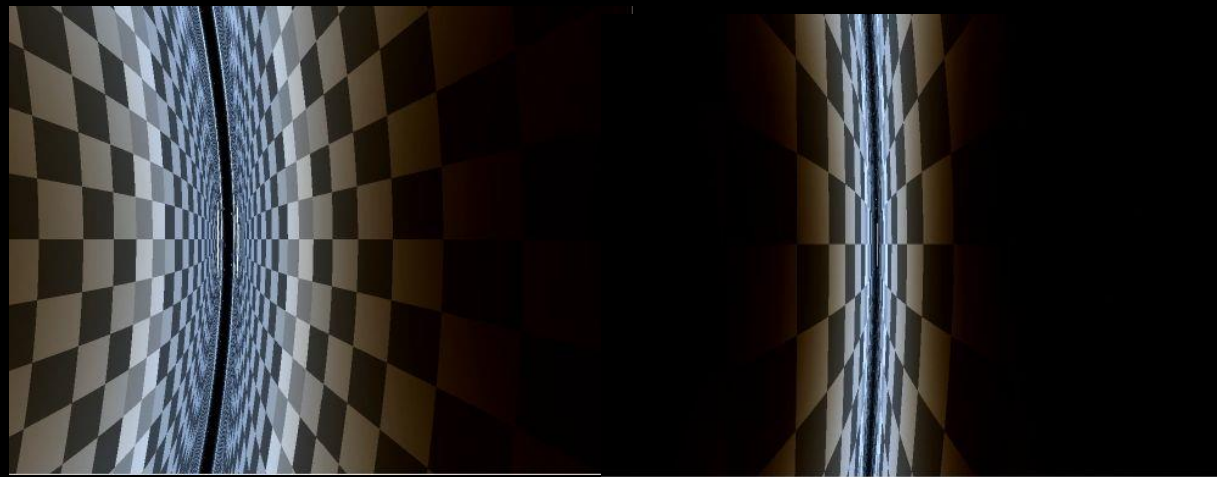
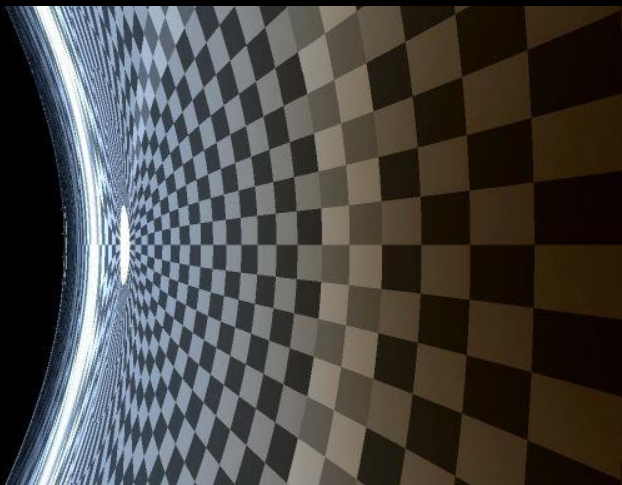
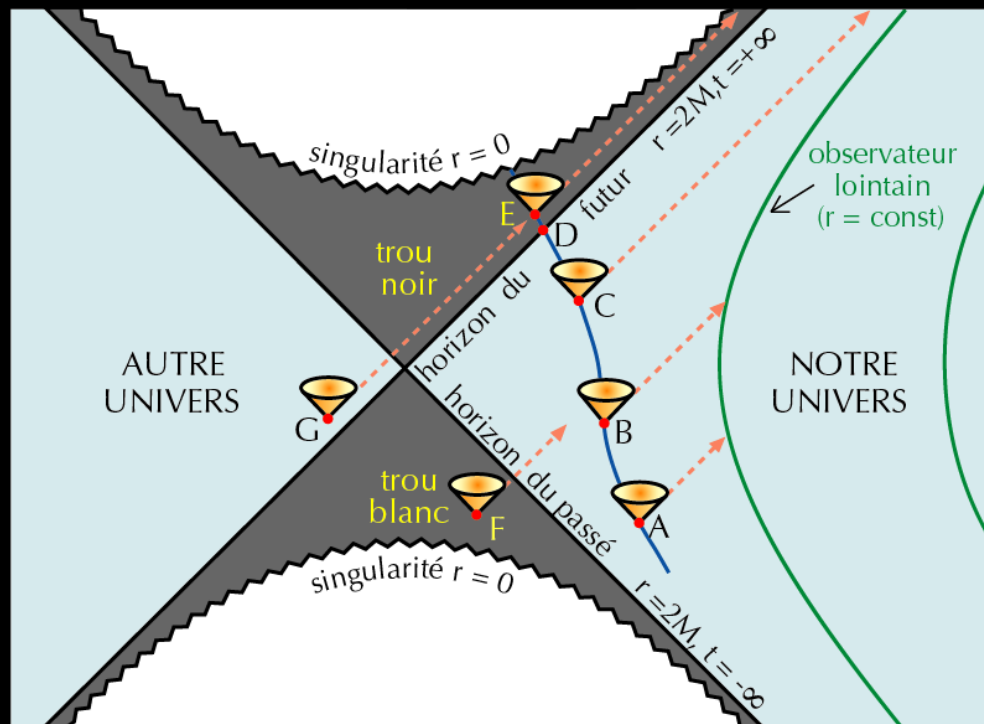


See movie 1

Flight into a static black hole

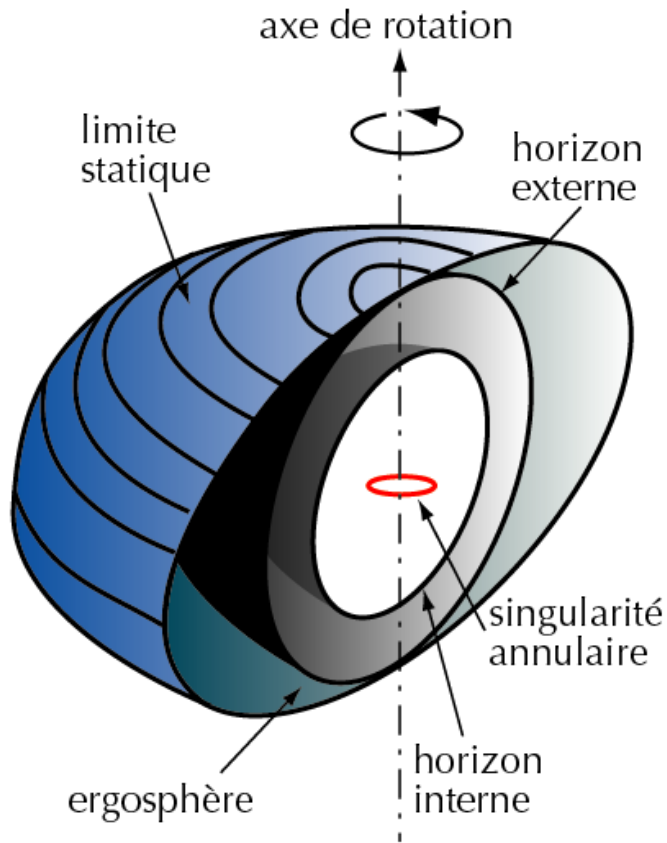
2

Non-radial photons

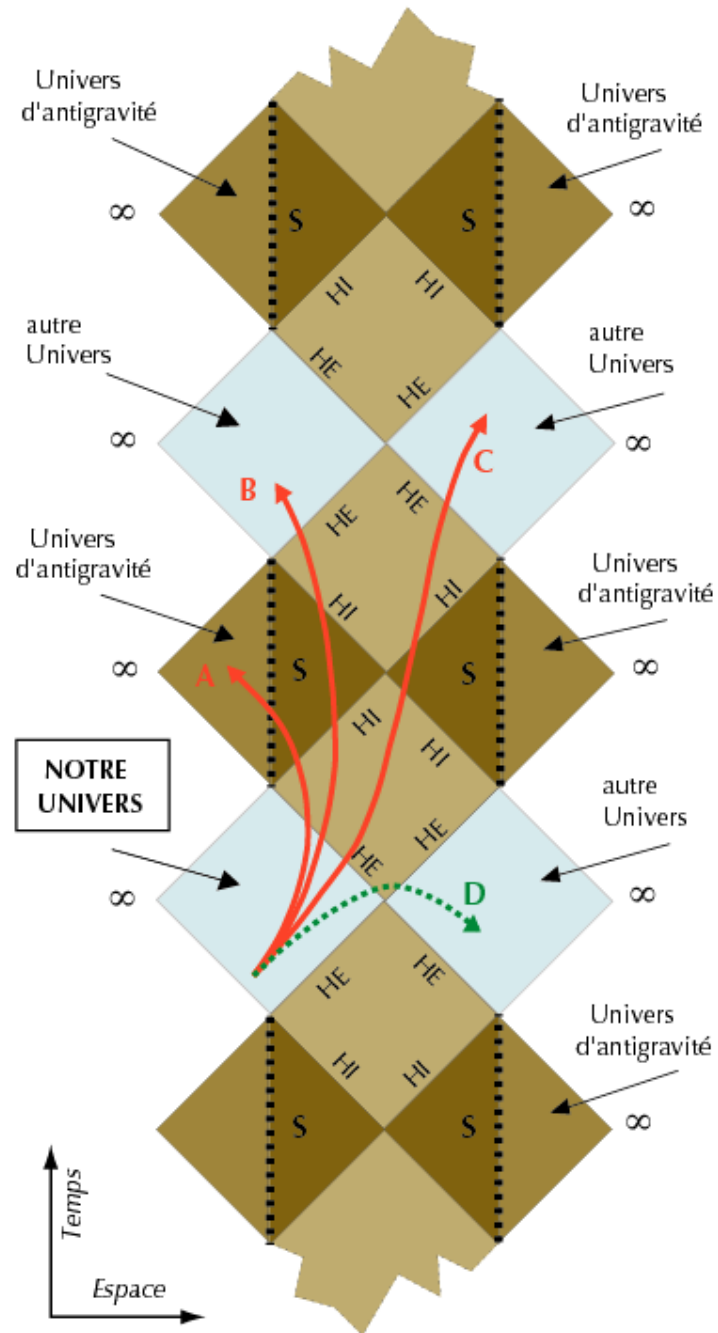


See movie 2

Flight into a Kerr (rotating) black hole



no movie yet!



Cosmology

A vast field of stars and galaxies, likely a deep sky image, serving as a background. The stars are of various colors, including yellow, white, blue, and red, and are scattered across the dark space. Some stars have prominent diffraction spikes. The word "Cosmology" is written in a red, stylized font at the top center of the image.

Homogeneity

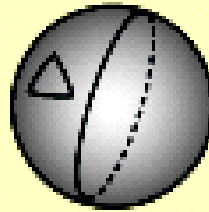
=>

constant
space
curvature !

$\Omega > 1$

courbure

$K > 0$

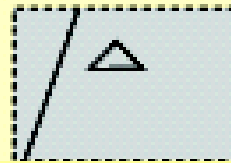


espace sphérique

finite (no edge)

$\Omega = 1$

$K = 0$

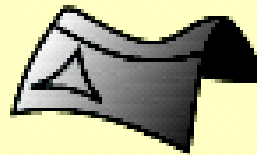


espace Euclidien

finite or
infinite

$\Omega < 1$

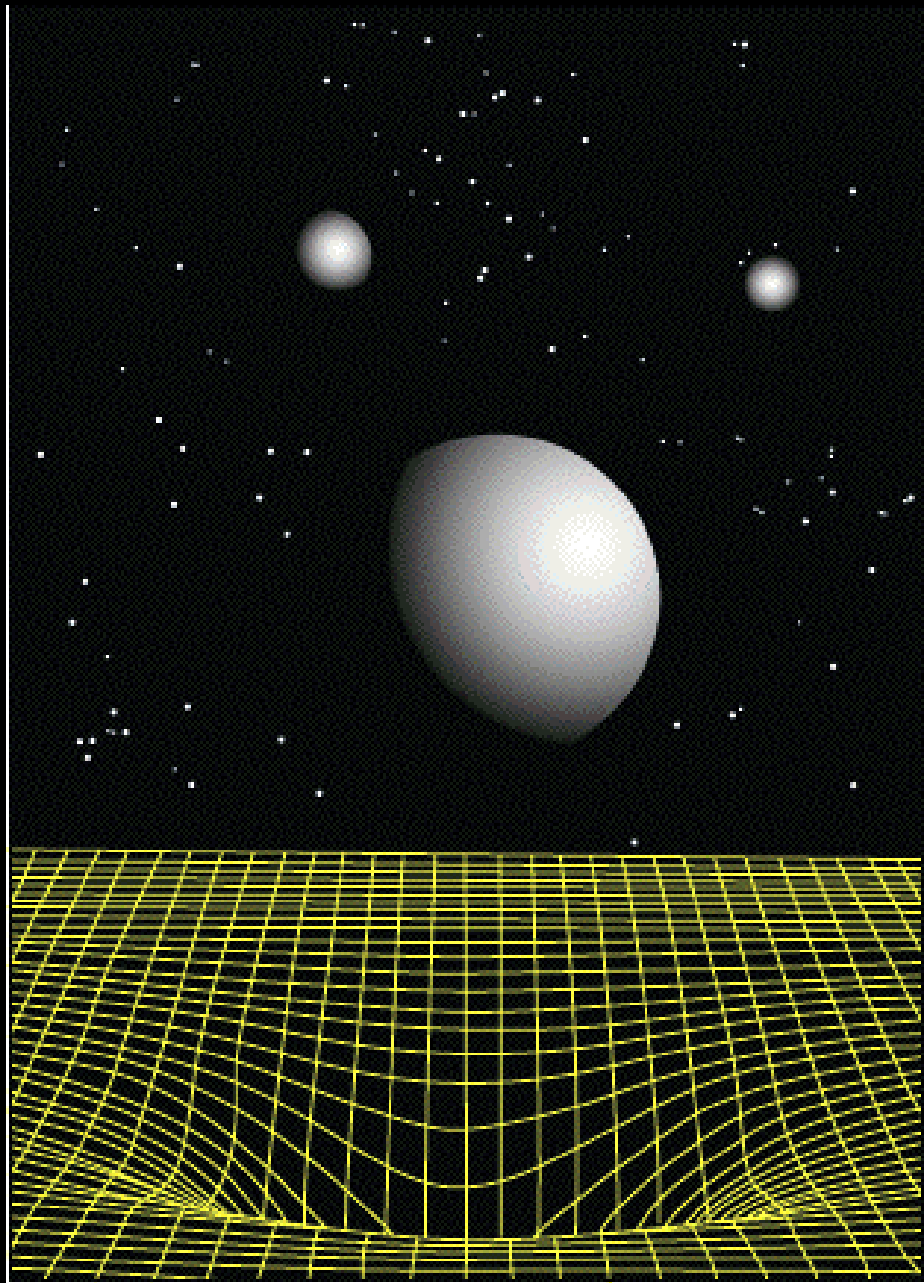
$K < 0$



espace hyperbolique

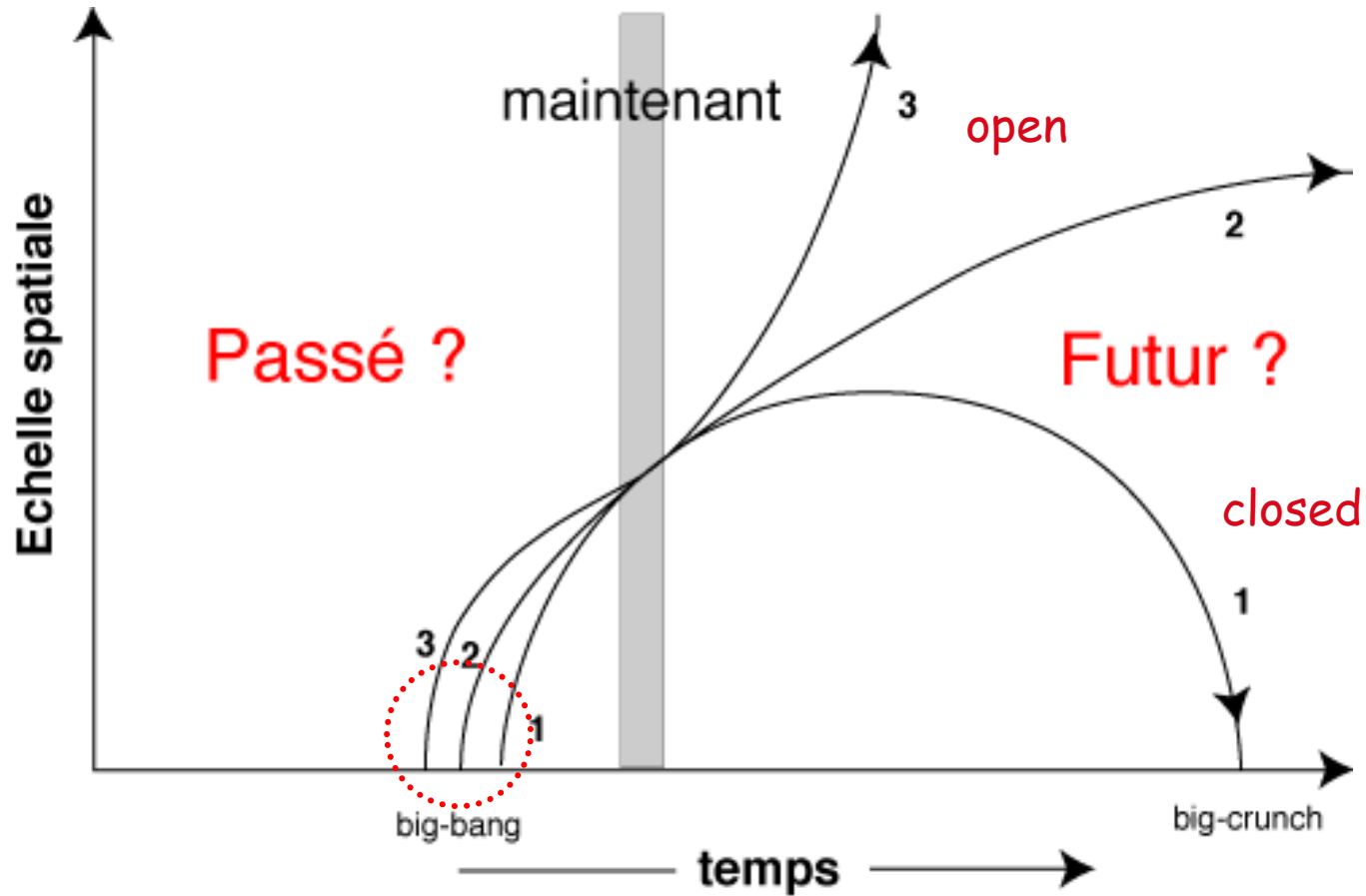
finite or
infinite

Space-time
curvature ==>
a dynamical
universe !

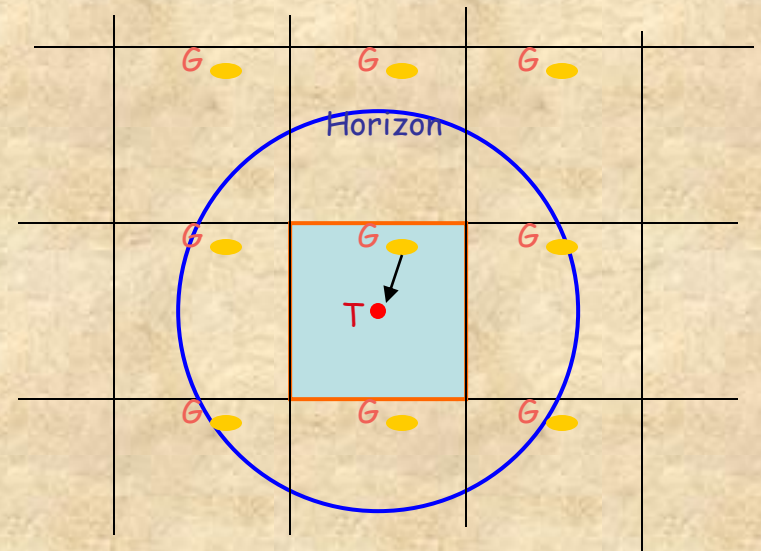
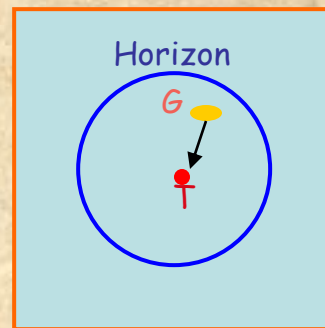
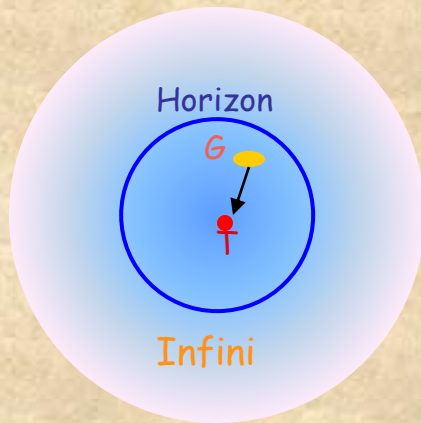


Expansion

Big bang models



What is the size and shape of space ?



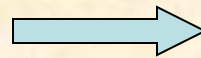
Not testable
(only $L \gg R_h$)

May be testable ·
if $L \gtrsim R_h$

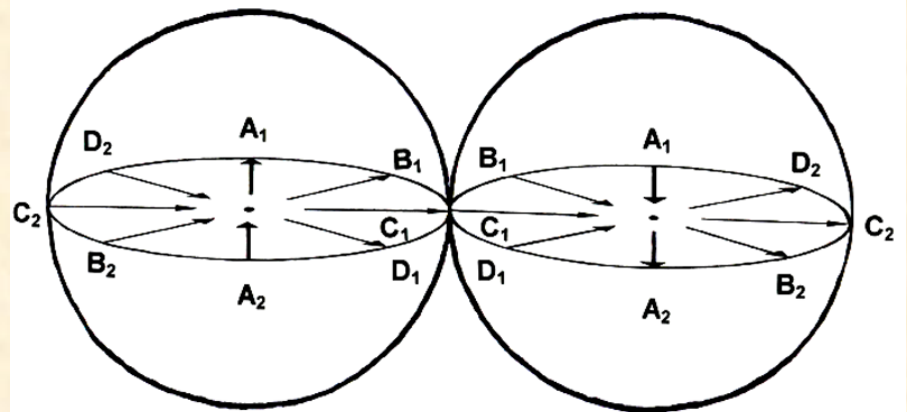
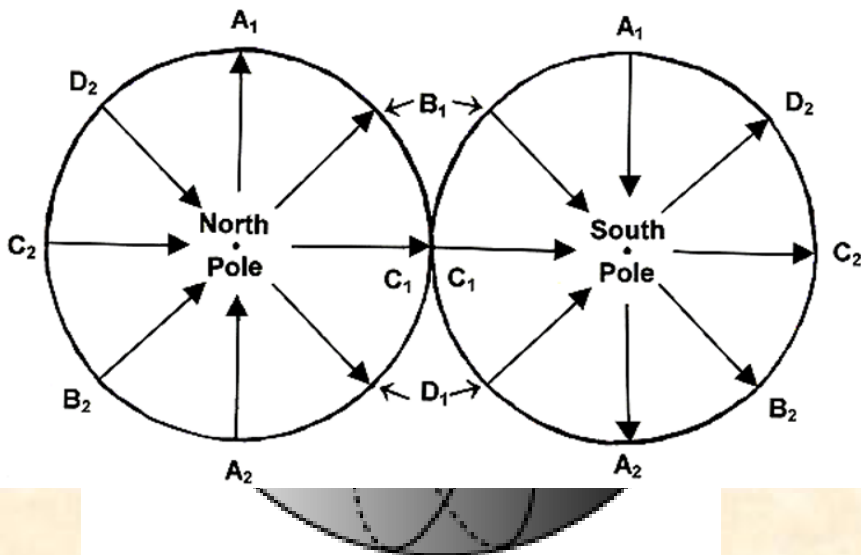
Testable
· topological lensing

Think finite space without edge

Sphere = 2D Surface
finite area, no edge

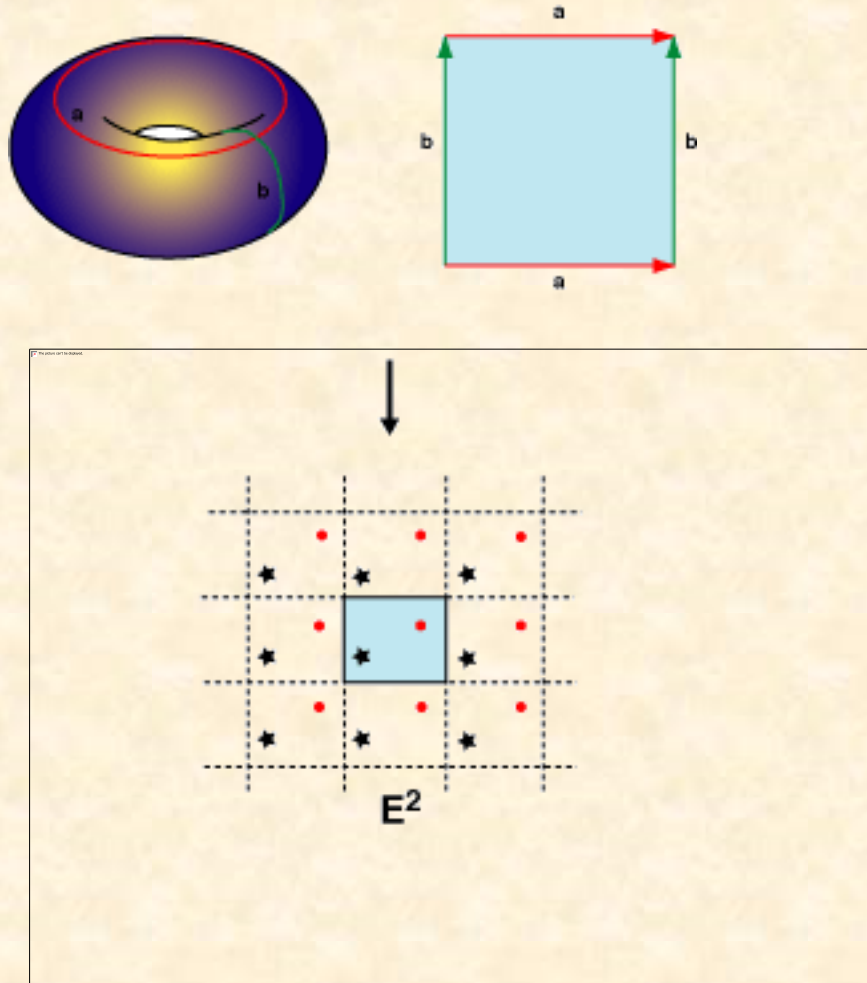


Hypersphere = 3D space
finite volume, no edge

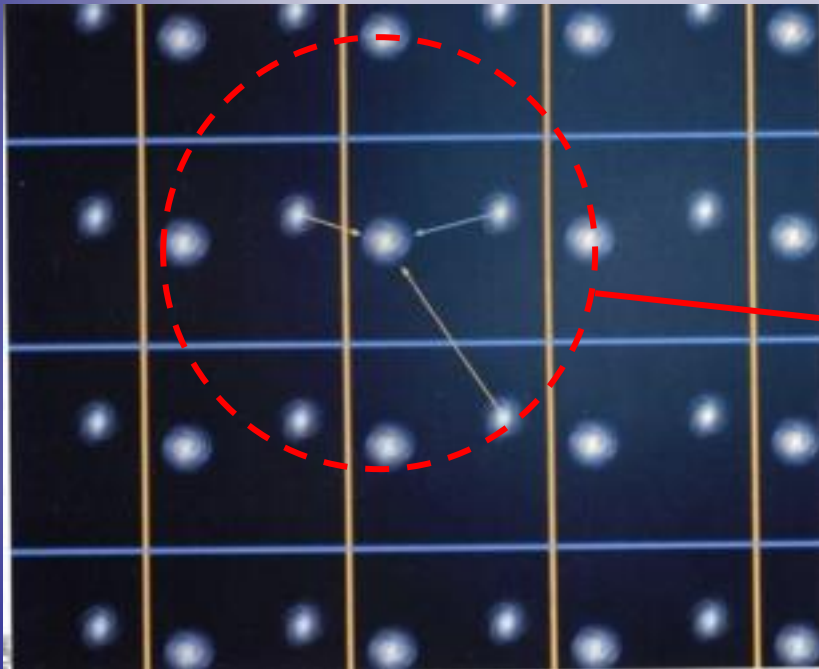
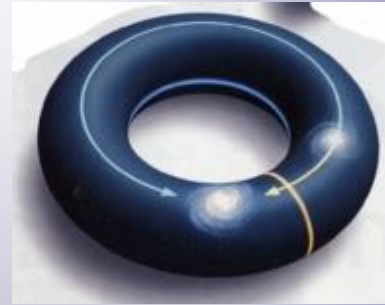
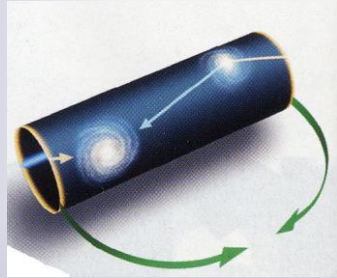
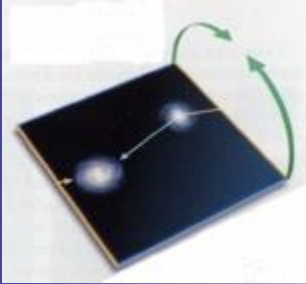


A finite flat space without a boundary

- Torus

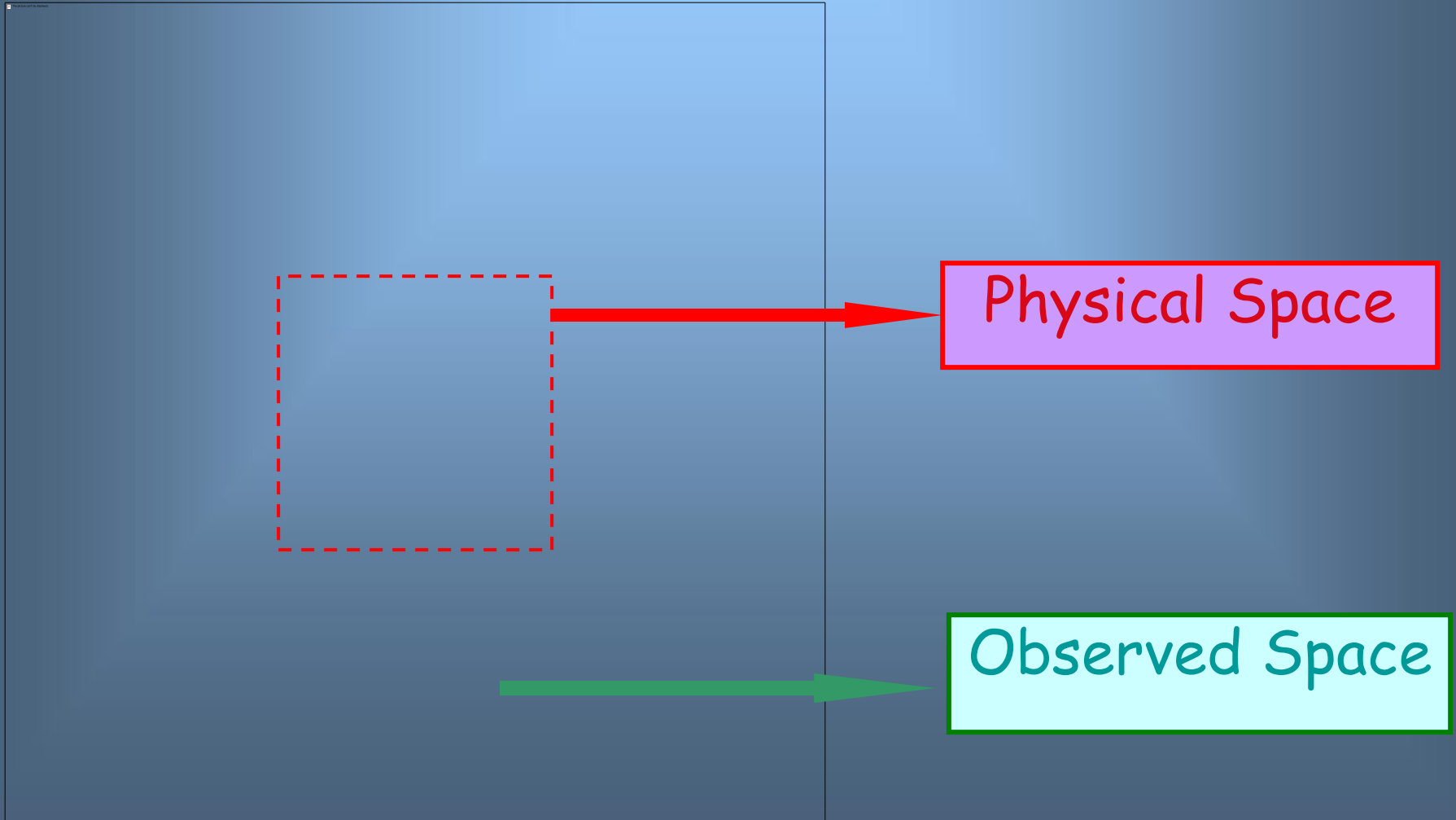


Topological lens effect

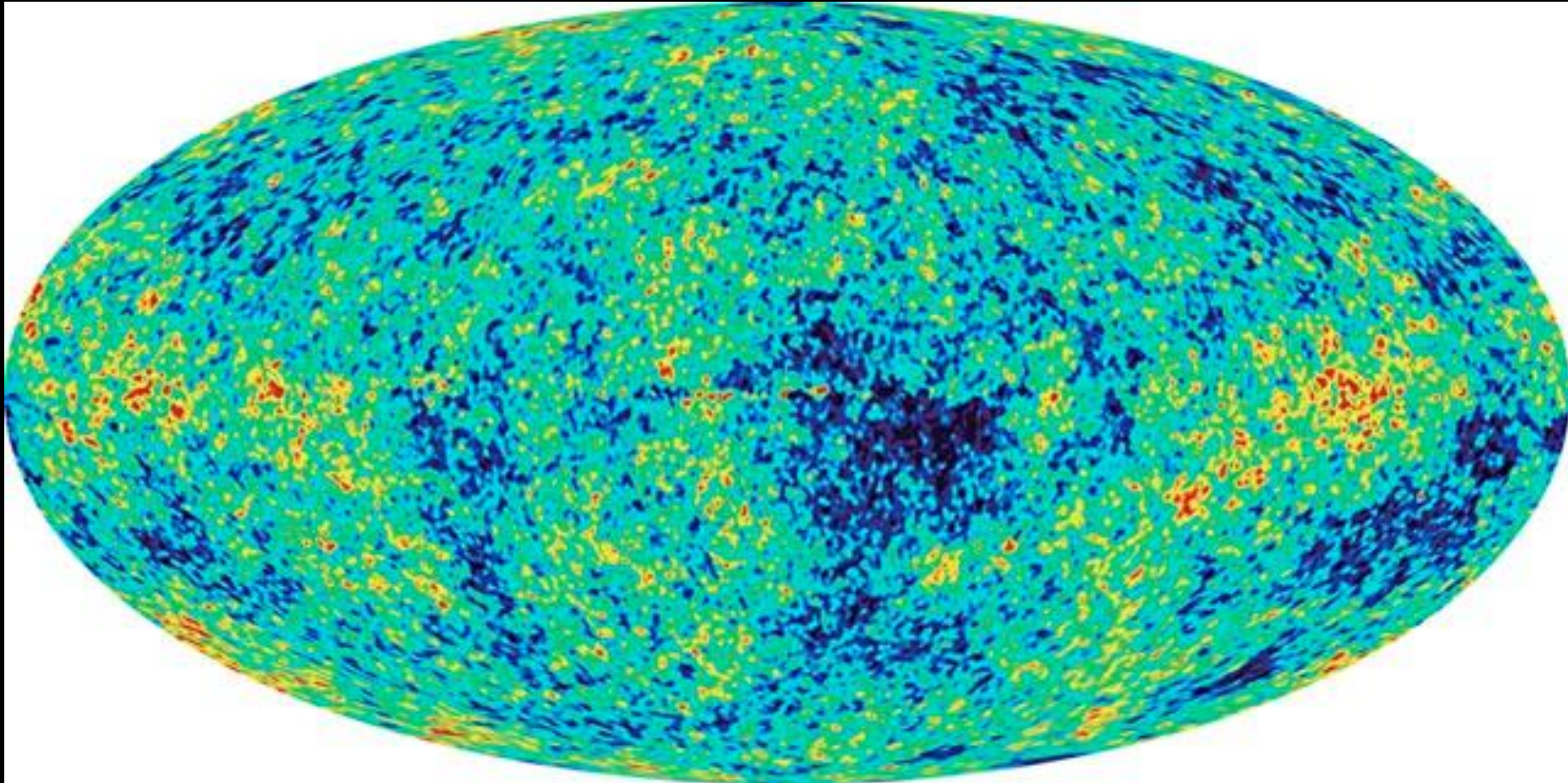


horizon

Hypertorus



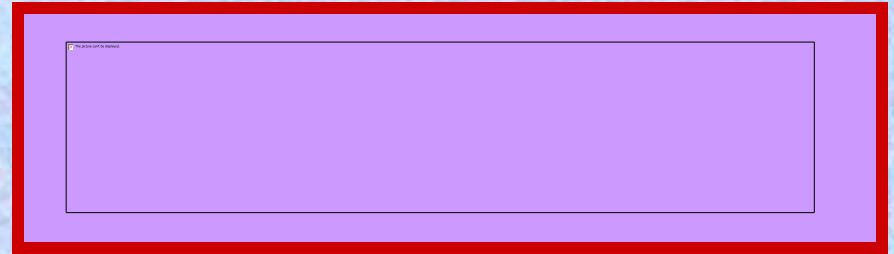
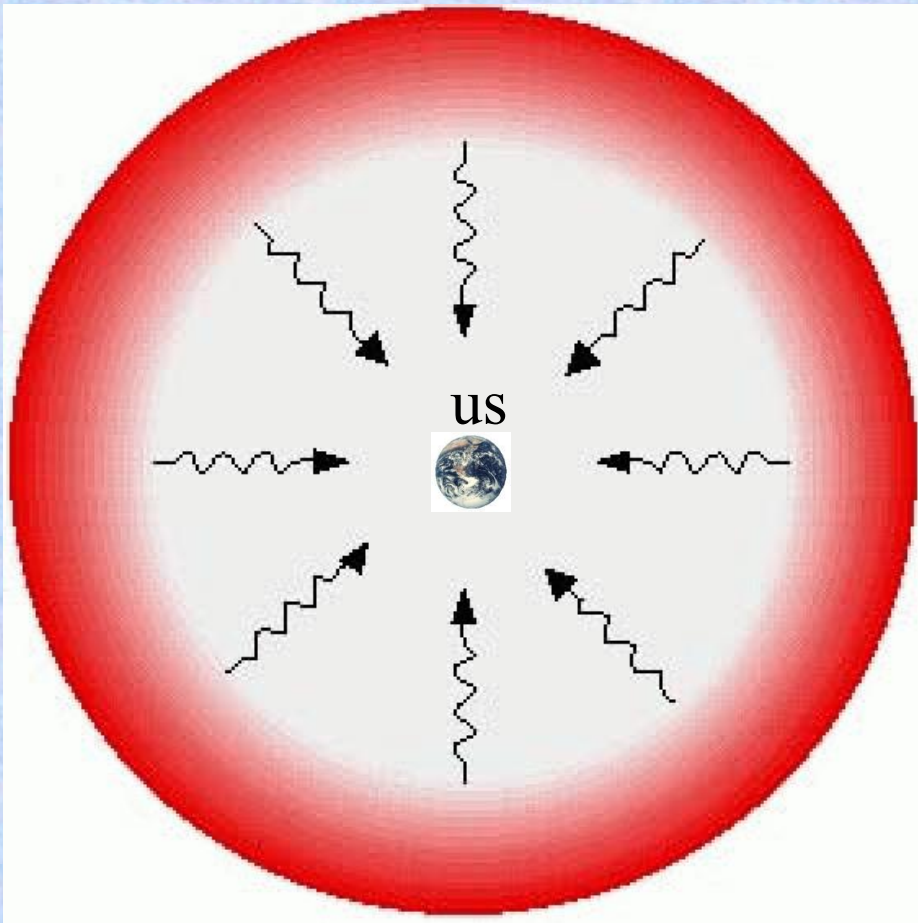
Cosmic Microwave Background



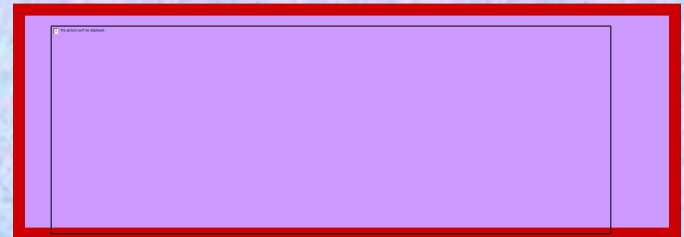
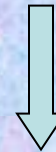
The universe as a cosmic « drumhead »

Cosmic Microwave Background

Observed on a 2-sphere



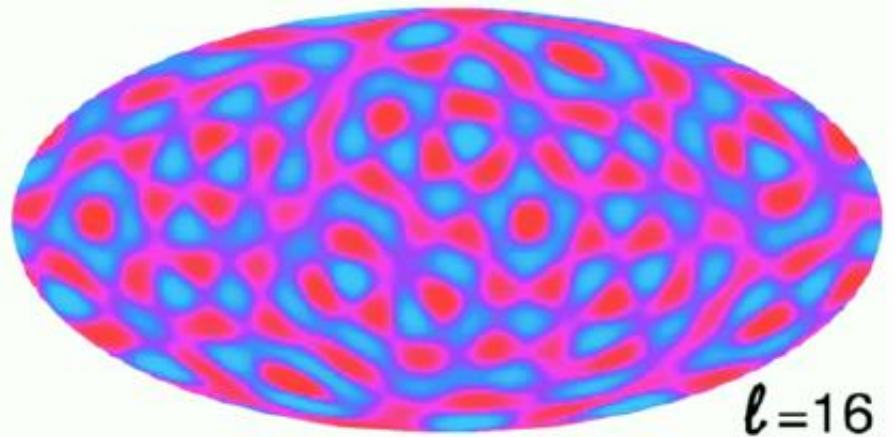
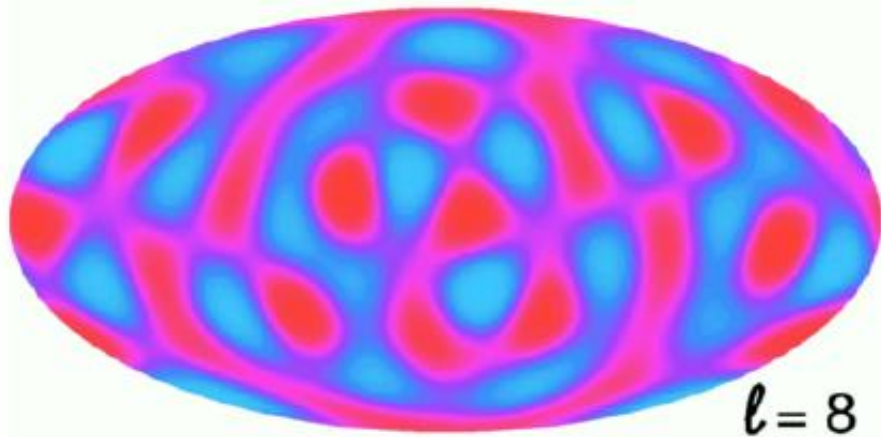
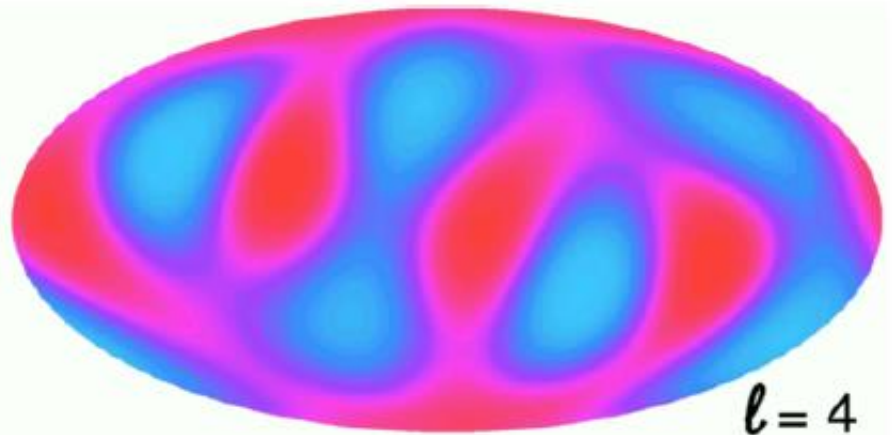
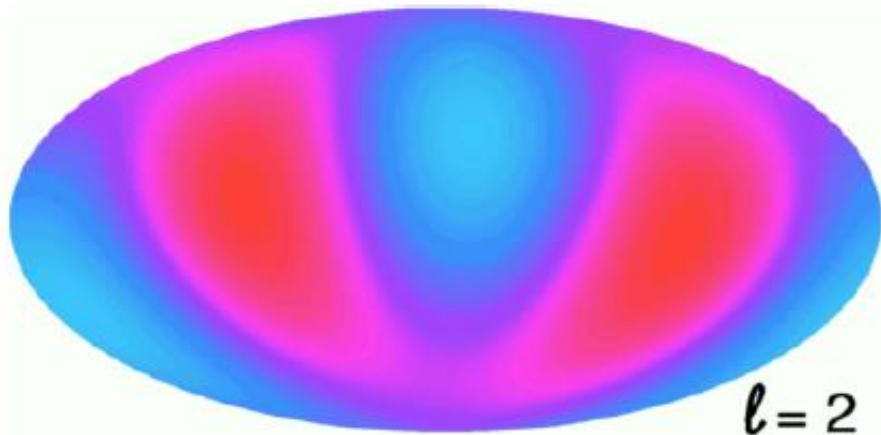
Spherical harmonics



Multipole moments

The CMB multipoles

Quadrupole

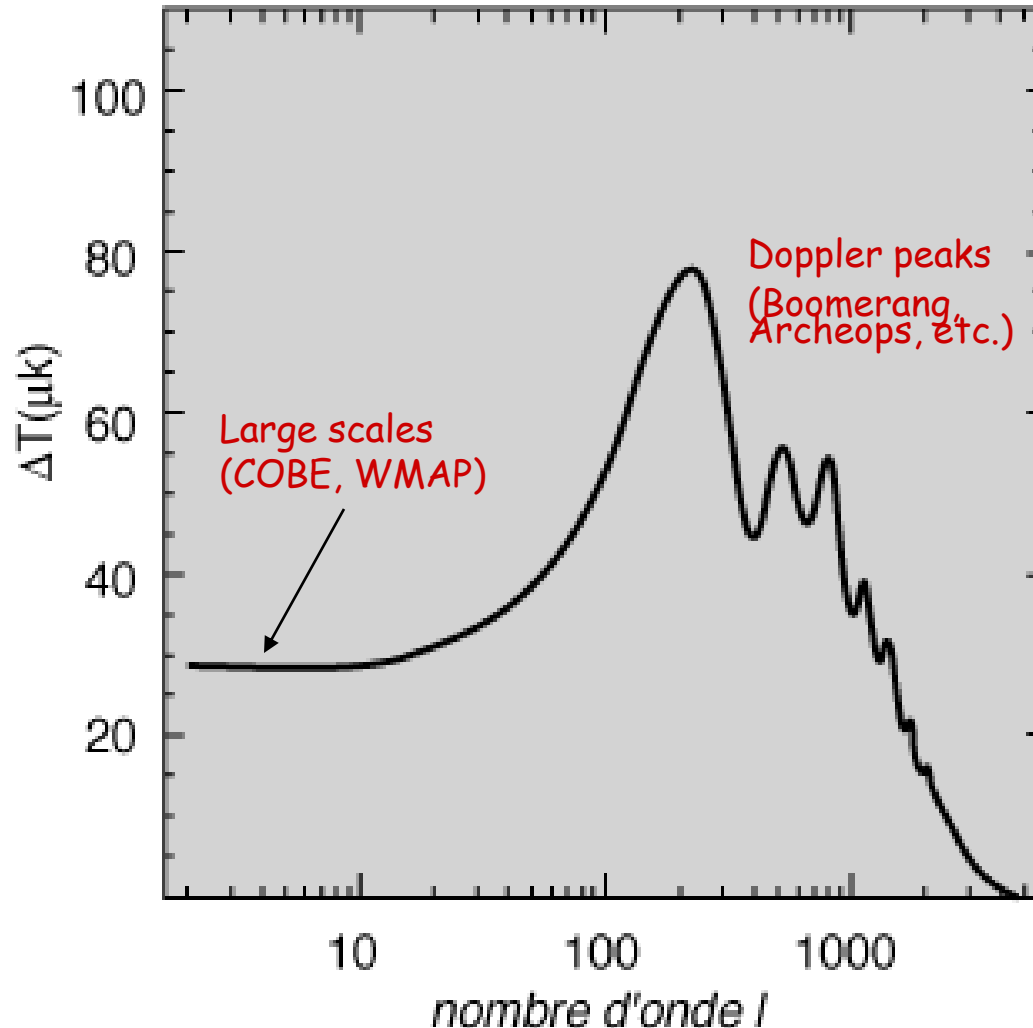


Power spectrum

$$\delta T_l^2 = l(l+1)C_l/2\pi$$

échelle angulaire

90° 20° 2° 0,2°



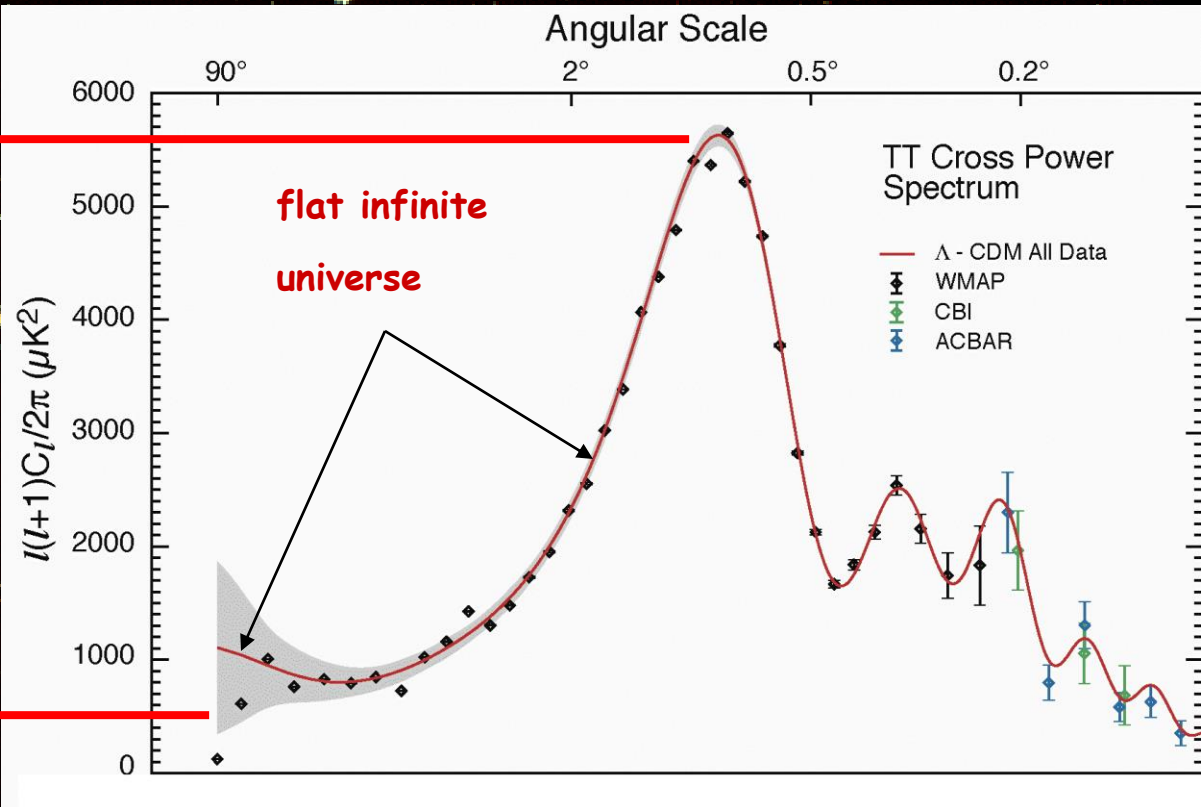
$$l = 180^\circ / \theta$$

WMAP power spectrum (2003- 2006)

- Universe seems to be positively curved

$$\Omega_{\text{total}} = 1.02 \pm 0.02$$

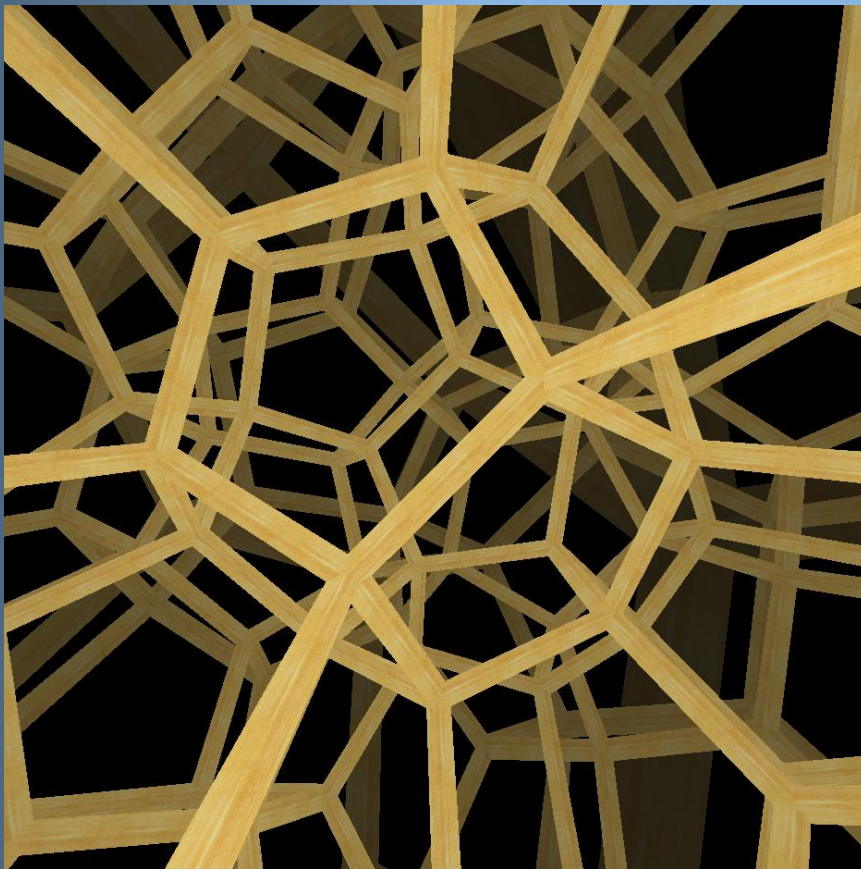
- Lack of power at large scales ($> 60^\circ$)



Space might be finite with a special shape!

Poincaré Dodecahedral Space

FP : 12 faces regular dodecahedron



S^3/I^*

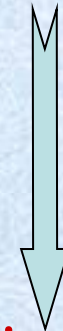
120 copies
tessellate S^3

Poincaré Dodecahedral Spherical space (PDS)



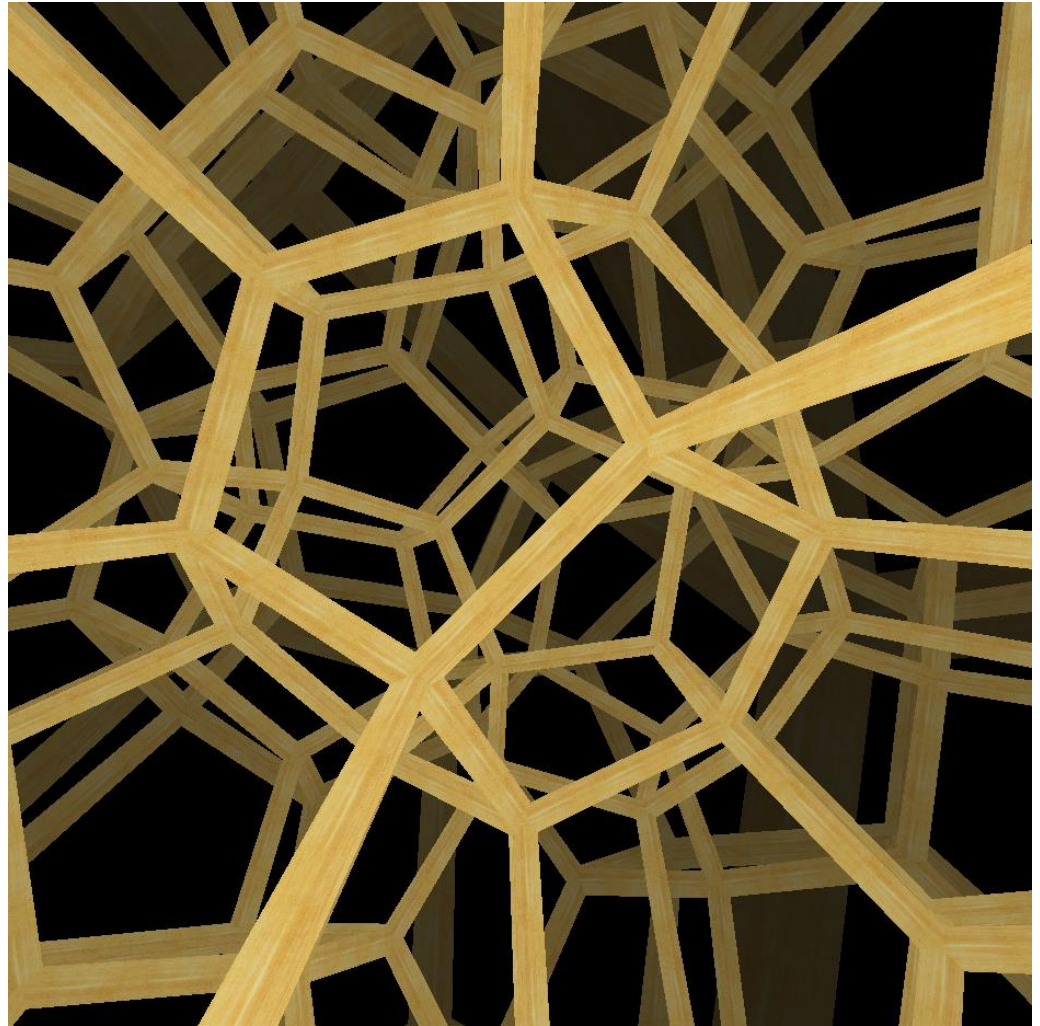
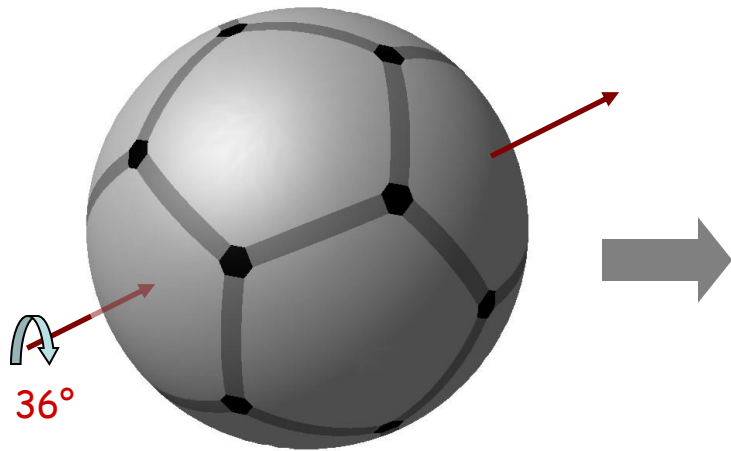
Luminet et al., *Nature* 425, 593
(2003)

- fit low quadrupole
- fit low octopole
- $0.9 < \Omega_{\text{tot}} < 1.02$

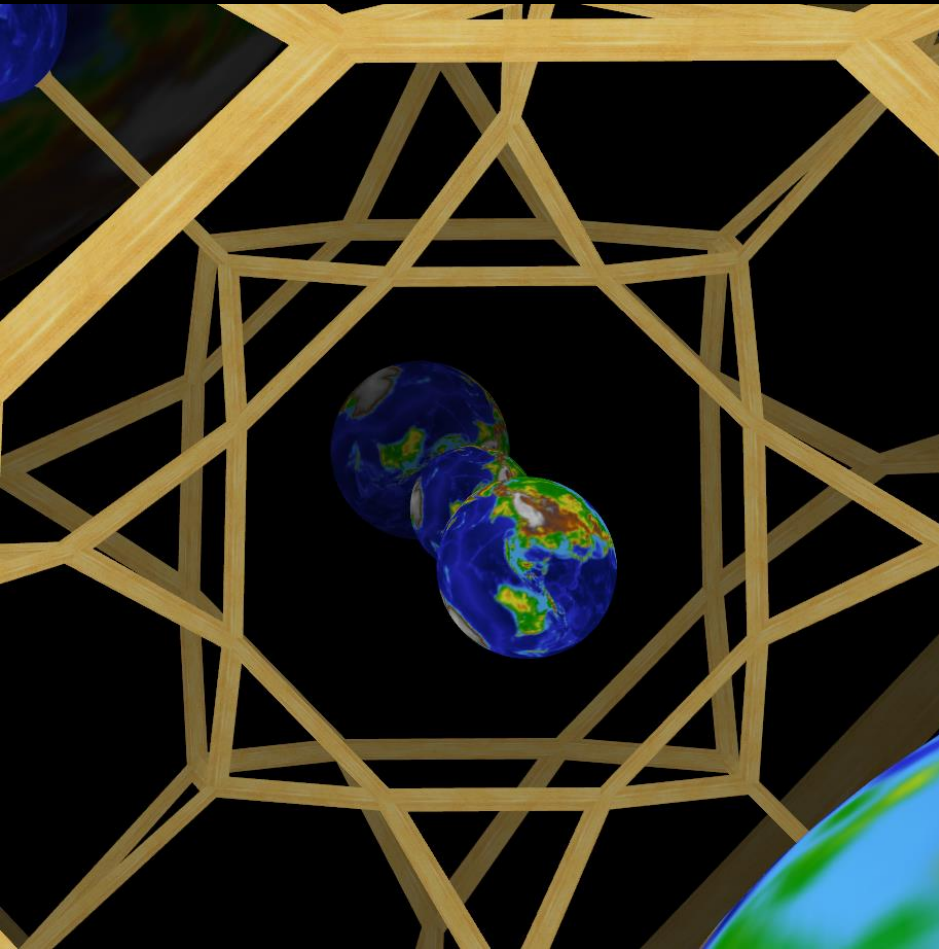


Planck Surveyor
(2007)

The « football Universe »

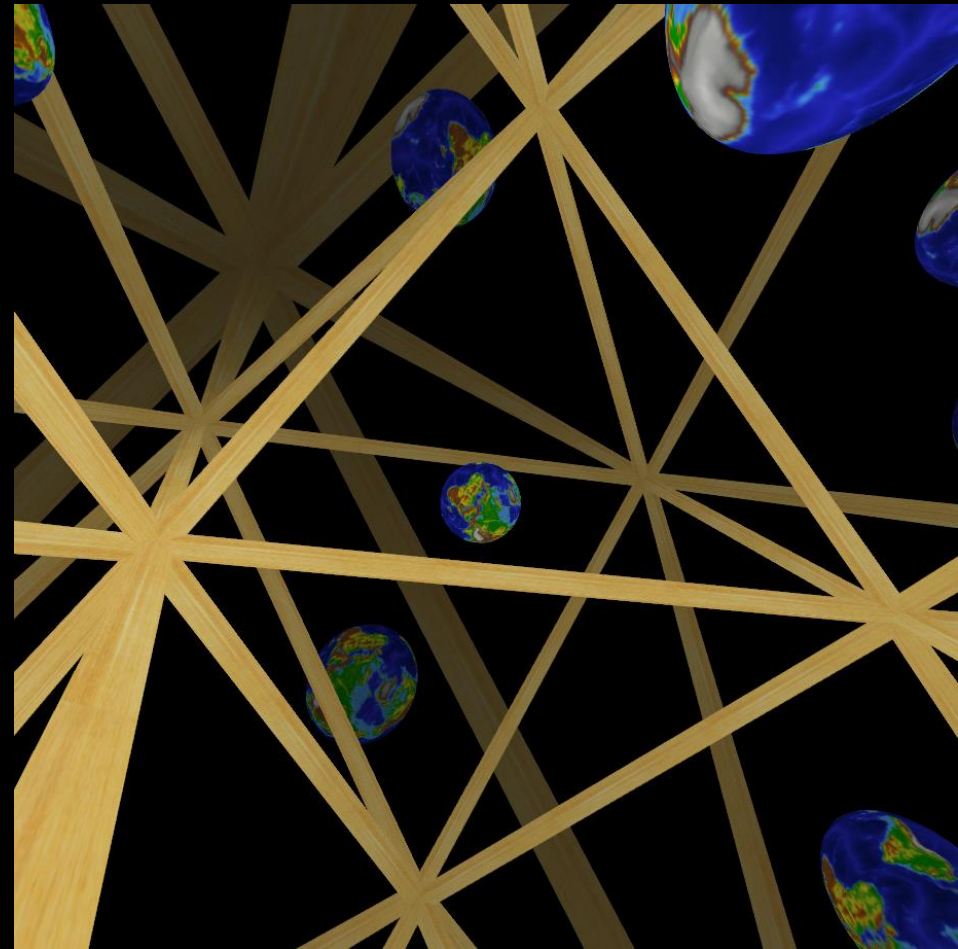


Also compatible ...



Octahedral
space

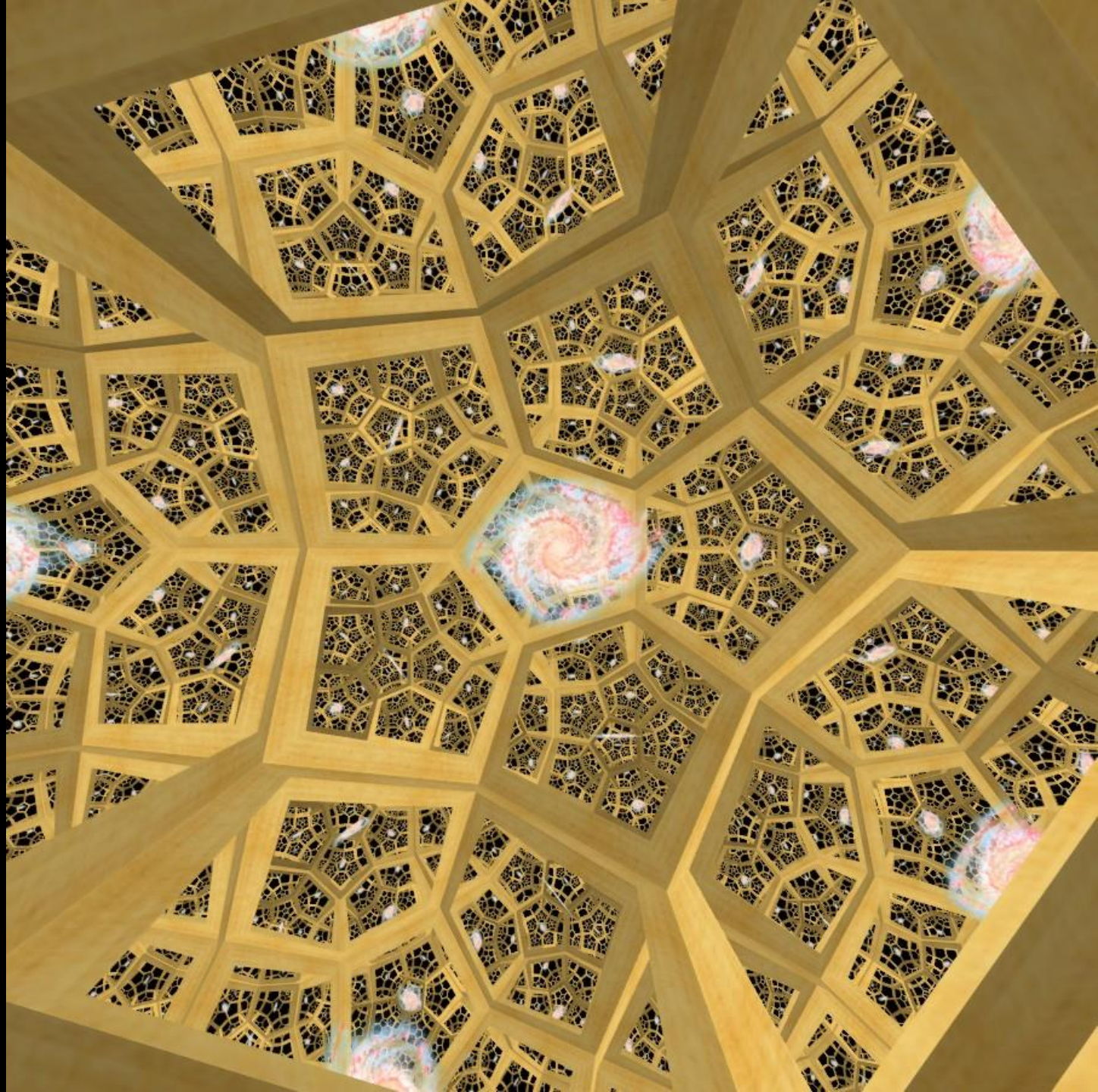
$$(\Omega_{\text{tot}} > 1.015)$$



Tetrahedral
space

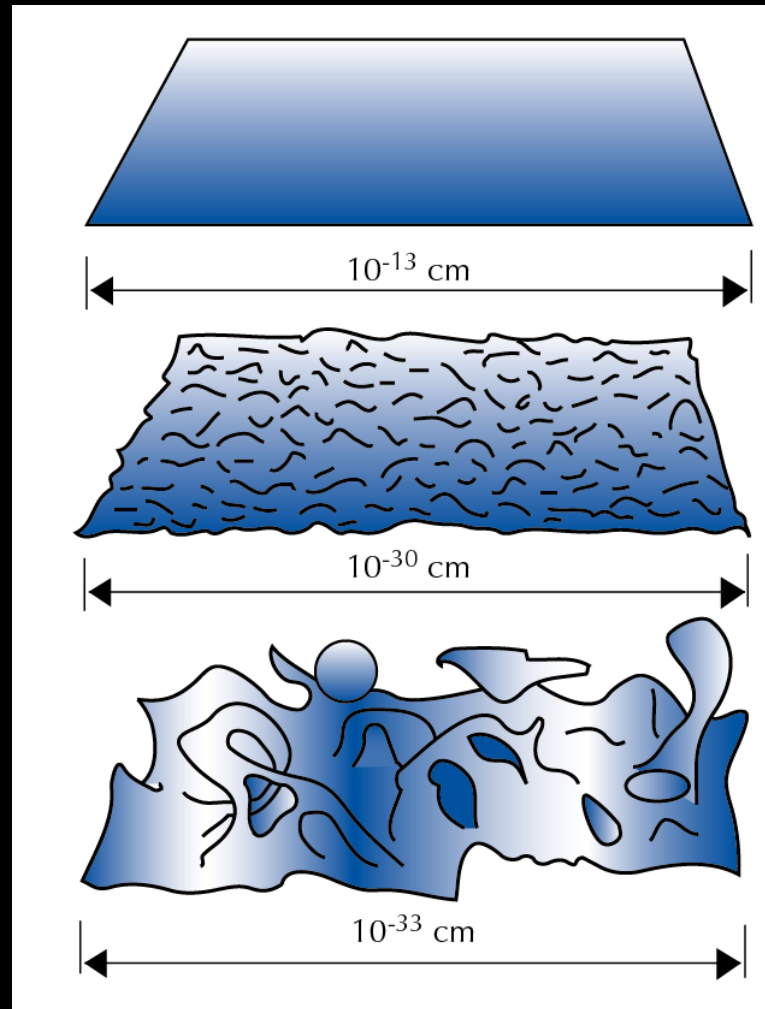
$$(\Omega_{\text{tot}} > 1.025)$$

J. Weeks, 2006



Imaging Quantum Gravity

Quantum foam
(J. Wheeler)



Solution 1 : string theory

Veneziano, Green,
Schwarz, Witten, etc.

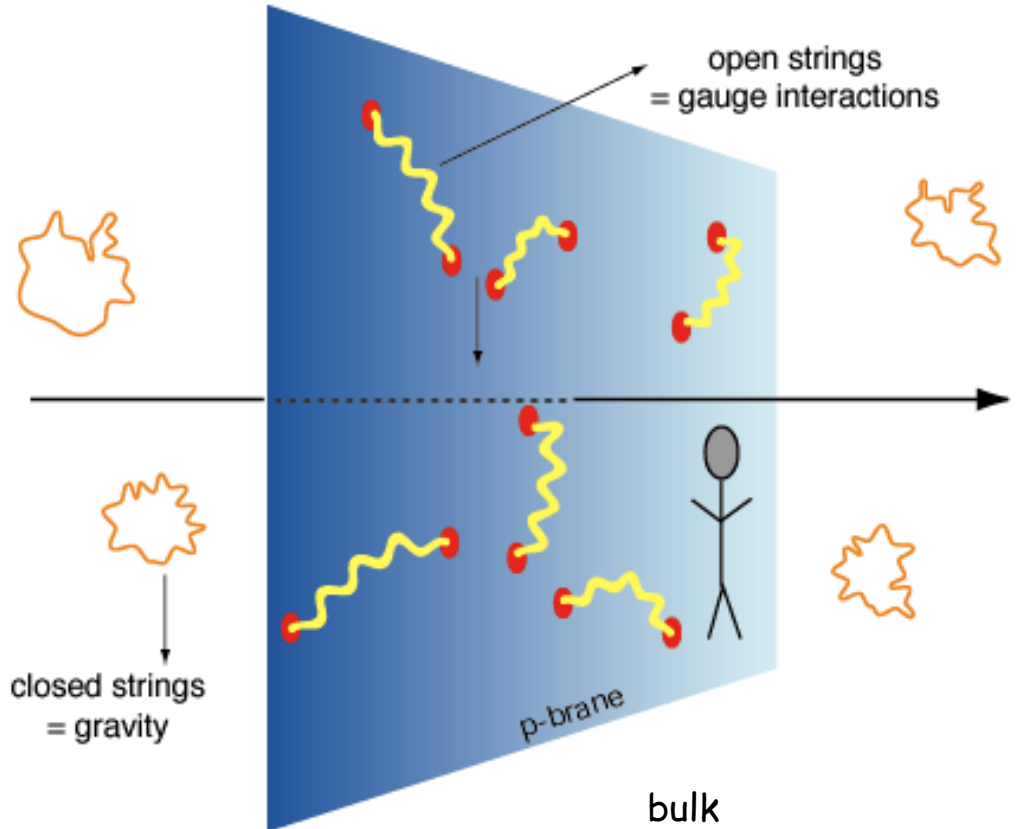
Price to pay :
extra-dimensions



Closed string



Open string



Solution 2 : loop quantum gravity

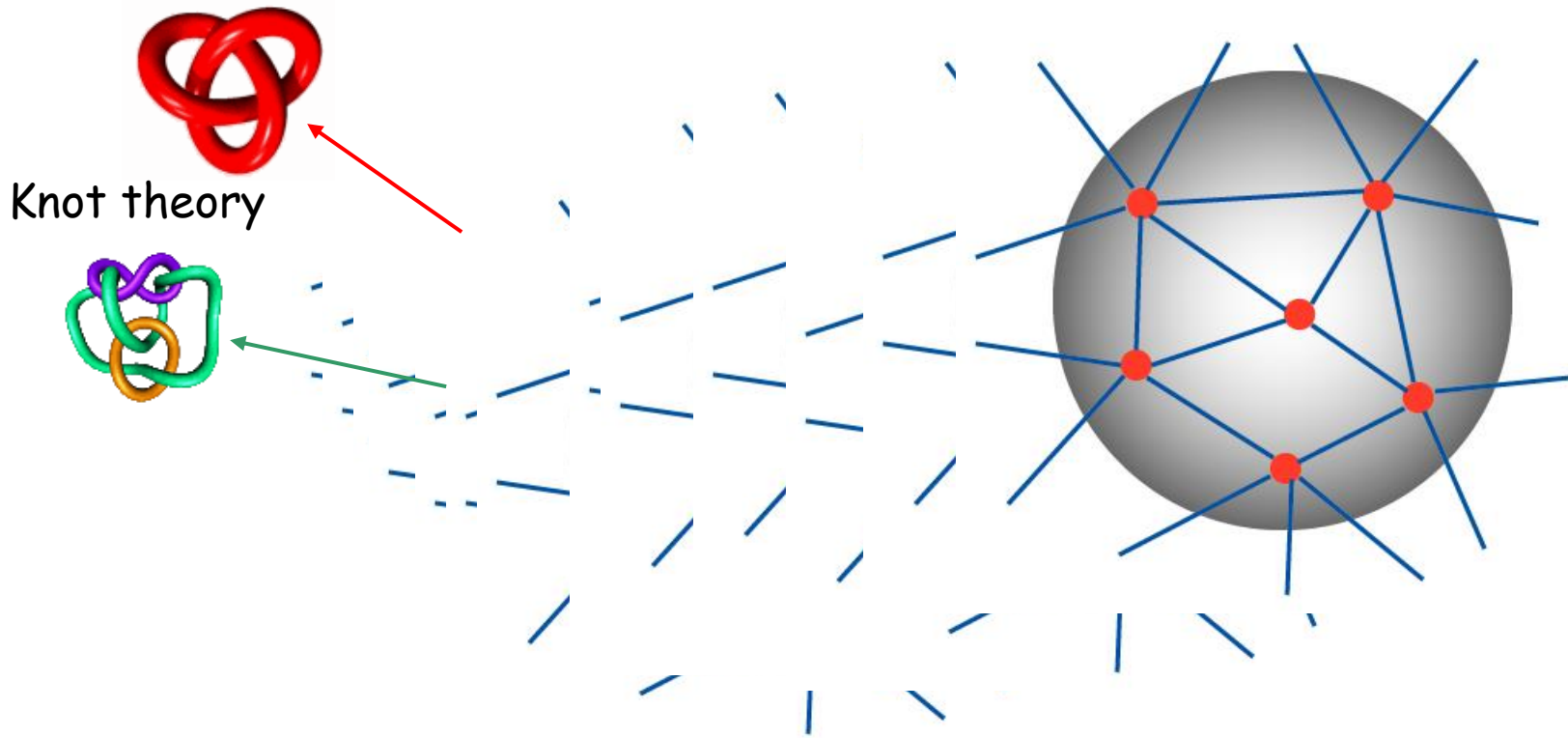
Ashtekhar, Smolin, Rovelli, Bojowald

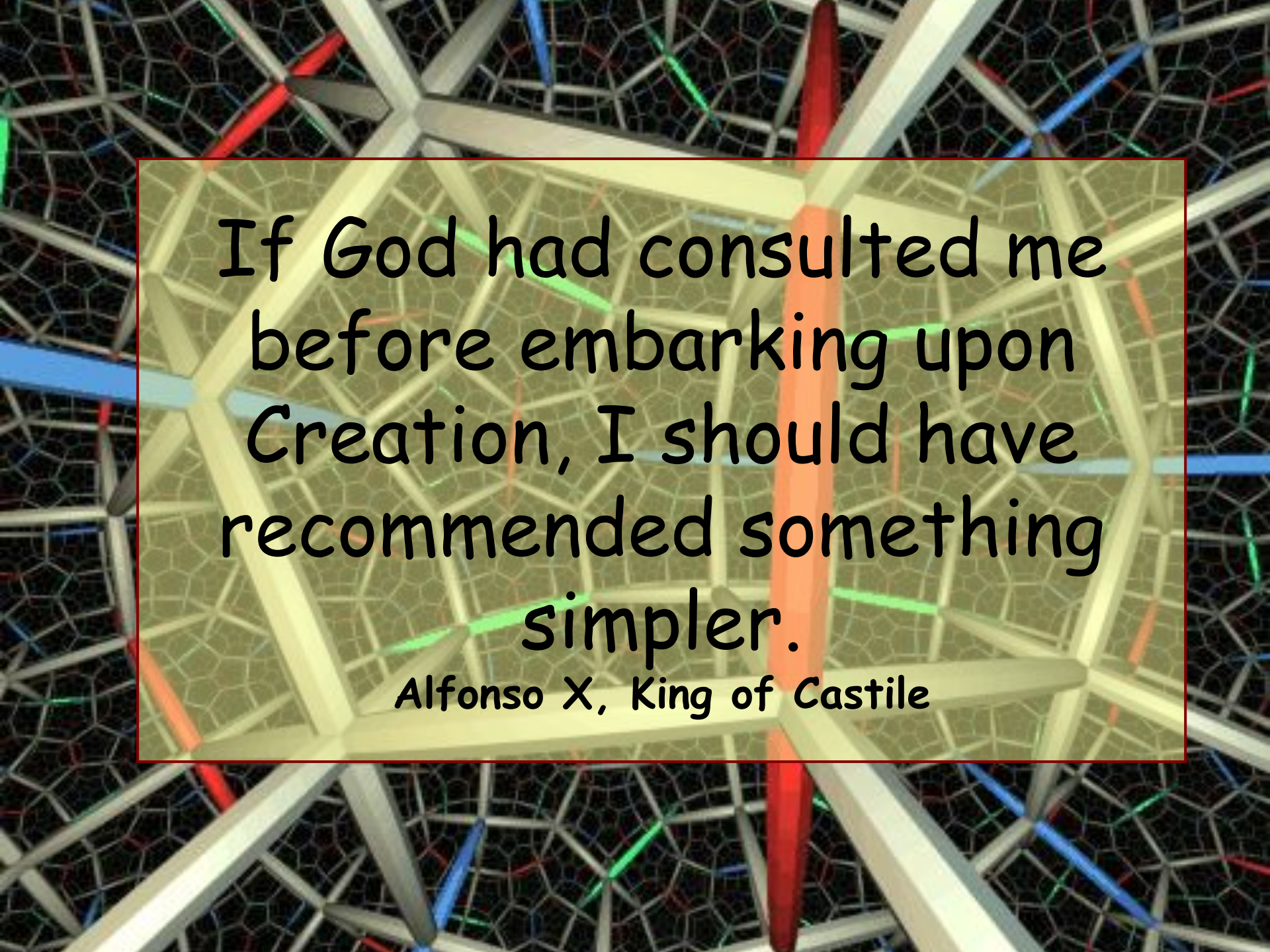
Atoms of space: 10^{-99} cm^3

Spin network

Atoms of time : 10^{-43} sec

Spin foam





If God had consulted me
before embarking upon
Creation, I should have
recommended something
simpler.

Alfonso X, King of Castile