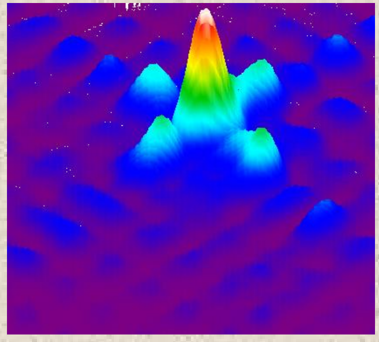
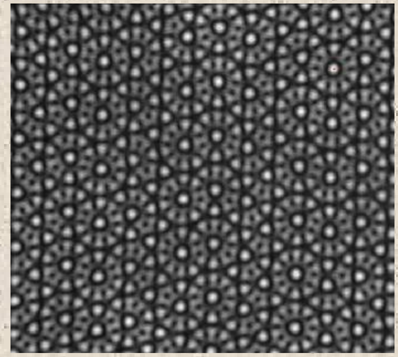
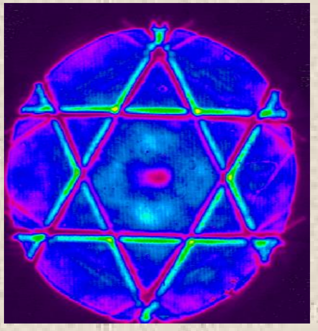
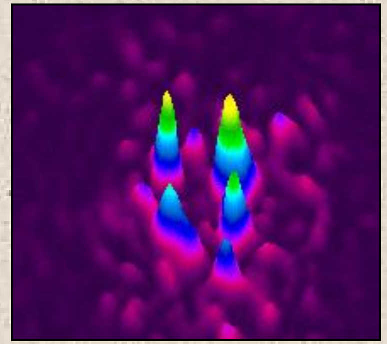
Solitons and Nonlinear Waves in Photonic Lattices

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Physics Department, *Technion – Israel Institute of Technology*

Former students / post-docs … now collaborators

Prof. Hrvoje Buljan (**now @ Univ. of Zagreb**)
Prof. Jason W. Fleischer (**now @ Princeton**)
Prof. Marin Soljacic (**now @ MIT**)
Dr. Oren Cohen (**now @ Univ. of Colorado**)
Dr. Tal Carmon (**now @ Caltech**)

Collaborators

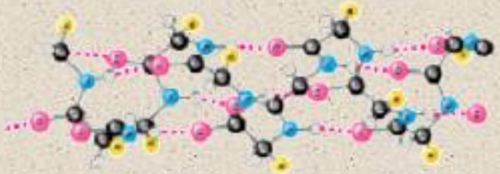
Prof. Demetri Christodoulides / **CREOL, UCF**
Prof. Ron Lifshitz / **Tel Aviv University**
Prof. Shmuel Fishman / **Technion**

Current students

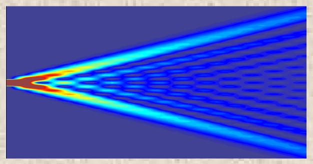
(almost Dr.) Guy Bartal / (almost @ Berkeley)
Barak Freedman
Ofer Manela
Tal Schwartz
Or Peleg

Outline

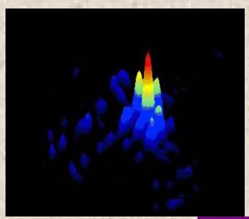
- Universality: nonlinear lattices in nature



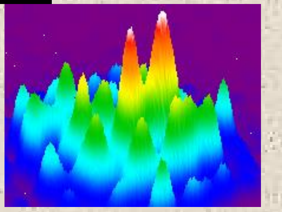
- Lattice transport and lattice solitons



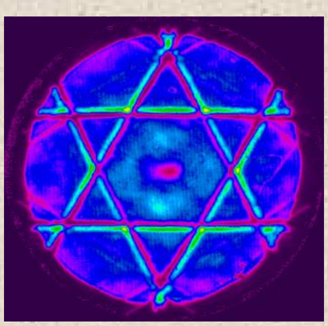
- 2D lattice solitons, vortices, etc.



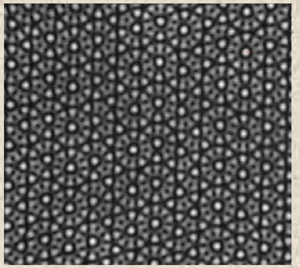
- Random-phase lattice solitons



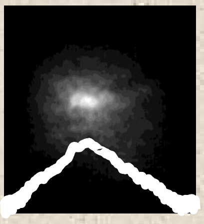
- Brillouin-zone spectroscopy of photonic lattices



- Nonlinear photonic quasi-crystals



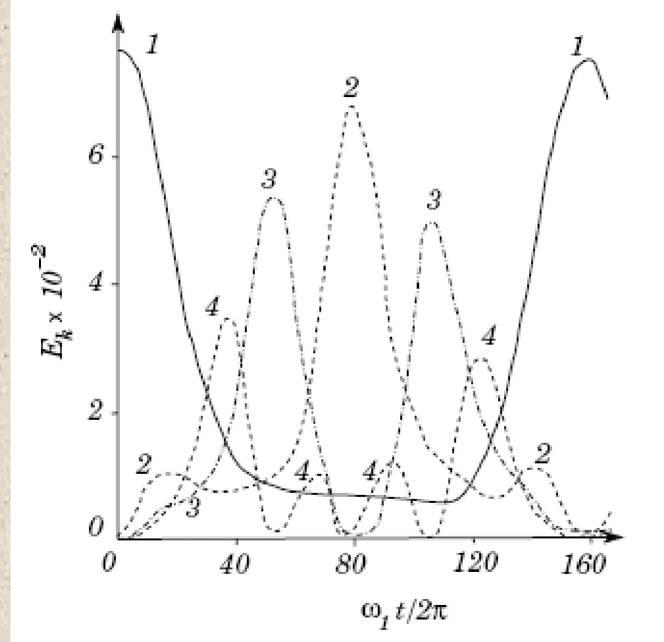
- Anderson localization



Fermi-Pasta-Ulam

Coupled anharmonic oscillators

Spring const = k - βx^2 => $F = -kx + \beta x^3$



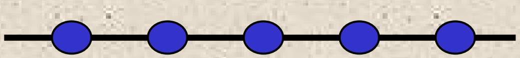
E. Fermi, J. Pasta, and S. Ulam
Los Alamos Report LA-1940 (1955).

First “numerical experiment”, ever

- Fermi-Pasta-Ulam looked for “thermalization”. **Surprise** – they got **recurrence!**
- Major step in the solution of the “FPU paradox” was obtained with the discovery that “solitons”, self-localized modes of nonlinear system, can interact elastically [Zabusky and Kruskal PRL 15 240 (1965)].
- When modes of a nonlinear system that do not exchange energy => **recurrence**

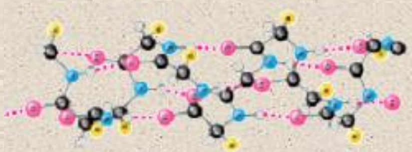
Nonlinear Lattices in Science

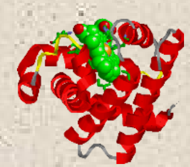
Coupled anharmonic oscillators



E. Fermi, J. Pasta, and S. Ulam
Los Alamos Report LA-1940 (1955).

Biology: phonon energy in α-helices



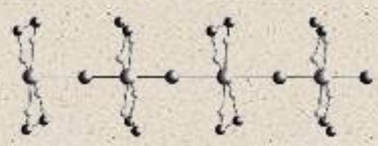


Myoglobin

A.S. Davydov,
J. Theor. Biol. **38**, 559 (1973).

A. Xie *et al.*,
PRL **84**, 5436 (2000).

Atomic chain



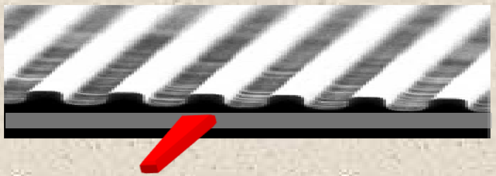
CDW waves in transition metals

B.I. Swanson *et al.*, PRL **82**, 3288 (2000).

Spin waves in antiferromagnets

U.T. Schwartz *et al.*, PRL **83**, 223 (1999).

Optics



Beam Envelope — Diffraction — Kerr NL — Periodic index

$$i\frac{\partial\psi}{\partial z}+\frac{1}{2}\nabla_{\perp}^{2}\psi+n_2|\psi|^2\psi+\Delta n_{Array}(r_{\perp})\psi=0$$

Nonlinear Schrödinger equation

Prediction:
D.N. Christodoulides *et al.*, Opt. Lett. 13, 794 (1988).

Experiment:
H. Eisenberg *et al.*, PRL 81, 3383 (1998).

Bose-Einstein Condensates

Gross-Pitaevskii (NLS) equation

Pair-wise scattering — Periodic potential

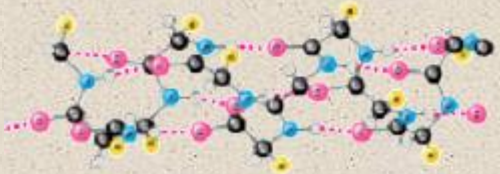
$$i\hbar\frac{\partial\psi}{\partial t}=-\frac{\hbar^2}{2m}\nabla^2\psi+U_0|\psi|^2\psi+V(\vec{r})\psi$$

A. Trombettoni and A. Smerzi, PRL **86** 2353 (2001).

B. Eiermann et al., PRL **92**, 230401 (2004).

Outline

- Universality: nonlinear lattices in nature



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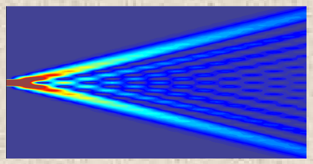

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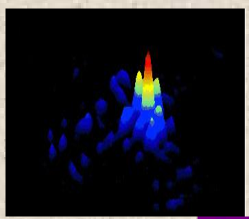
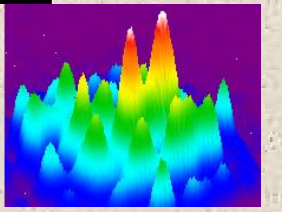
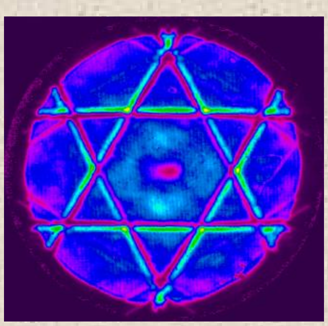


- Random-phase lattice solitons



- Brillouin-zone spectroscopy of photonic lattices



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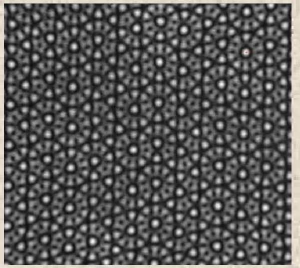

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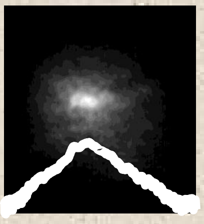
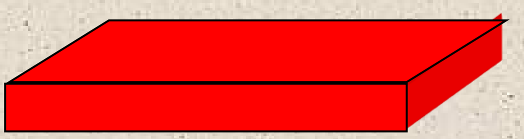
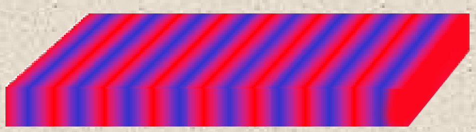


Linear Waves in Periodic Media

$$i\frac{\partial \psi}{\partial z} + \frac{\partial^2 \psi}{\partial x^2} + V(x)\,\psi = 0 \qquad \psi(x,z) = \Phi(x)e^{i\beta z}$$





Homogeneous: **V = 0**

Lattice: **V(x+d) = V(d)**

Translational symmetry

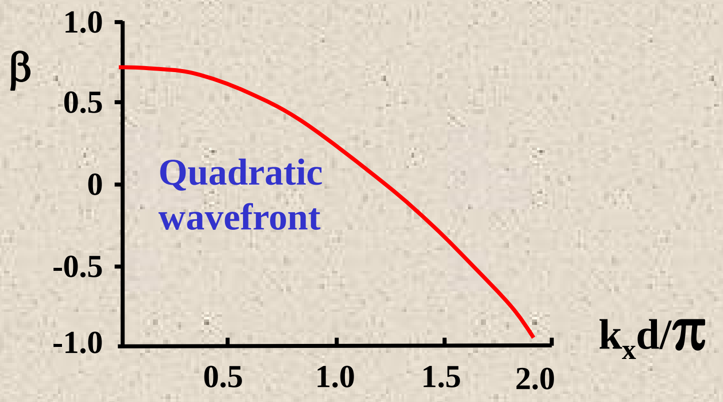
? ***Fourier basis*** $\left\{e^{ik_x x}\right\}$

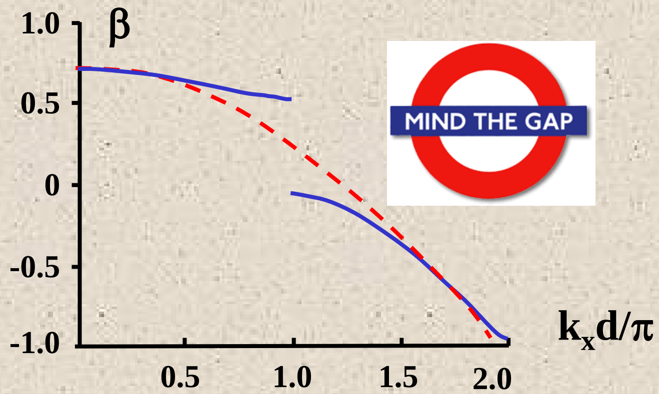
$[\ \beta \equiv \Delta k_z \cong -k_x^2/2k\ ]$

Periodicity

? ***Floquet-Bloch basis*** $\left\{u_{k_x}(x)e^{ik_x x}\right\}$

$$u_{k_x}(x+d) = u_{k_x}(x)$$





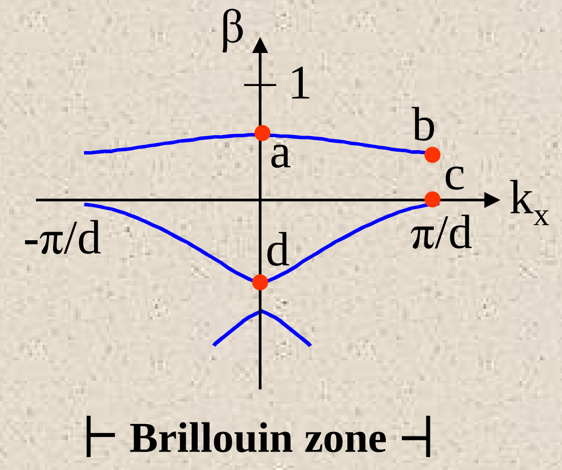
Linear Waves in Periodic Media

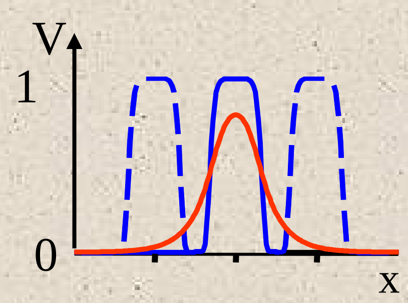
The modes (Floquet-Bloch waves) are extended waves

Transmission spectrum is divided into bands

Mode characterization:

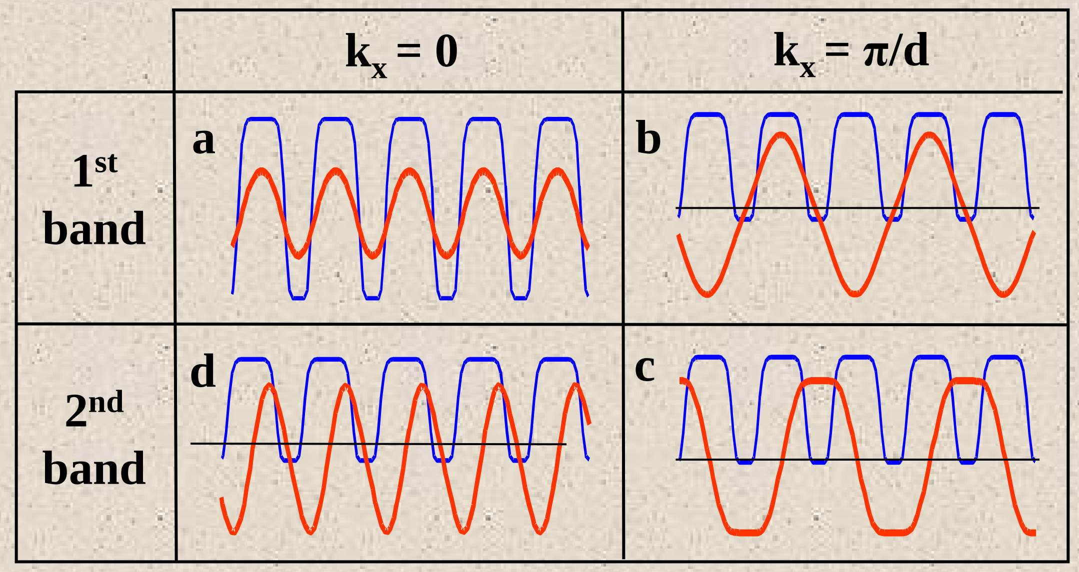
Band index and Bloch wavenumber k_x $\{-\pi/d < k_x < \pi/d\}$



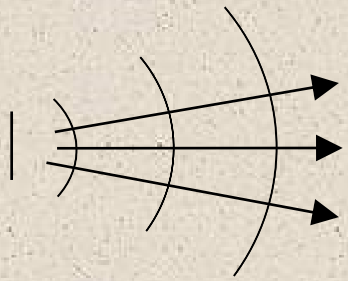


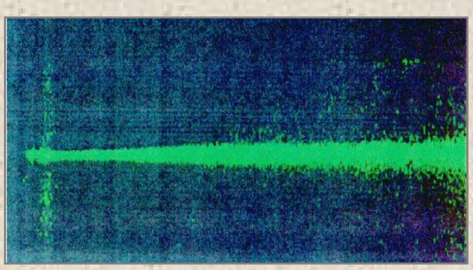
Consider array where each WG supports a single guided mode

FB modes of single-mode WG array



Linear Transport in Lattices





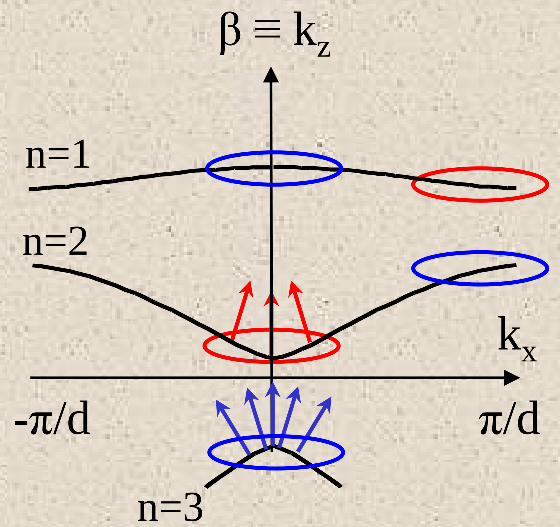
Homogeneous

Normal of wavefront defines direction of transport (ray):

$$\theta = \frac{\Delta x}{\Delta z} = \frac{\partial k_z}{\partial k_x}$$

Divergence of rays gives diffraction:

$$\frac{\partial^2 \psi}{\partial x^2} \leftrightarrow D \equiv \frac{\partial^2 k_z}{\partial k_x^2}$$

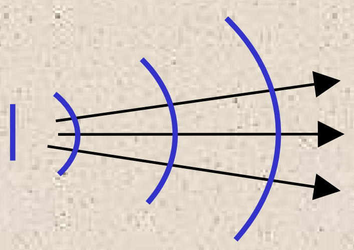


Periodic

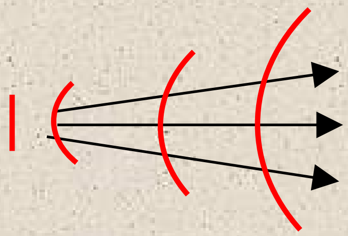
Normal Diffraction

Anomalous

Eisenberg *et al.*, PRL **85**, 1863 (2000).

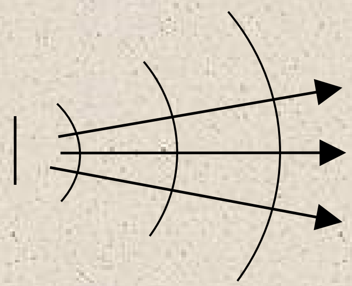


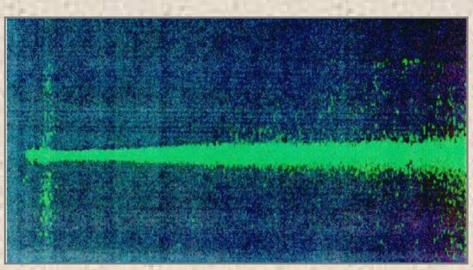
Normal diffraction (D<0)



Anomalous diffraction (D>0)

Linear Transport in Lattices





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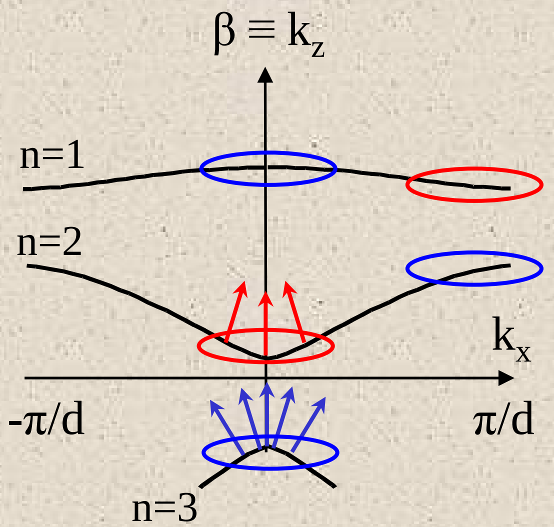
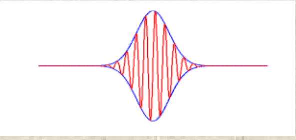


Periodic

Diffraction coefficient in lattices is = (effective mass)$^{-1}$

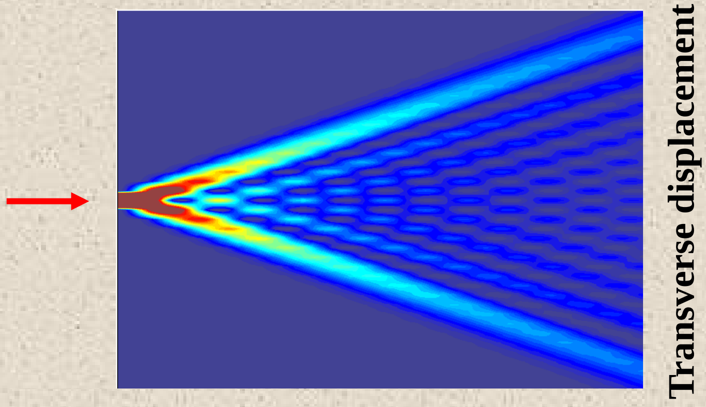
$$m_{eff}^{-1} \equiv \frac{1}{\hbar^2}\frac{\partial^2 \varepsilon}{\partial k^2}$$



Discrete / Lattice Solitons

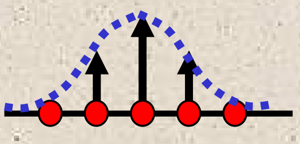
Predicted: Christodoulides 1988 ; First observed: Eisenberg et al., 1998

Linear transport in a lattice



Non-linear transport in a lattice

$$i\frac{d\psi}{dz}+\frac{1}{2}\frac{\partial^2\psi}{\partial x^2}+\gamma|\psi|^2\psi=0$$



Linear diffraction:

A.L. Jones, (1965)
Garmire & Yariv (1973)

Nonlinear propagation & discrete solitons:

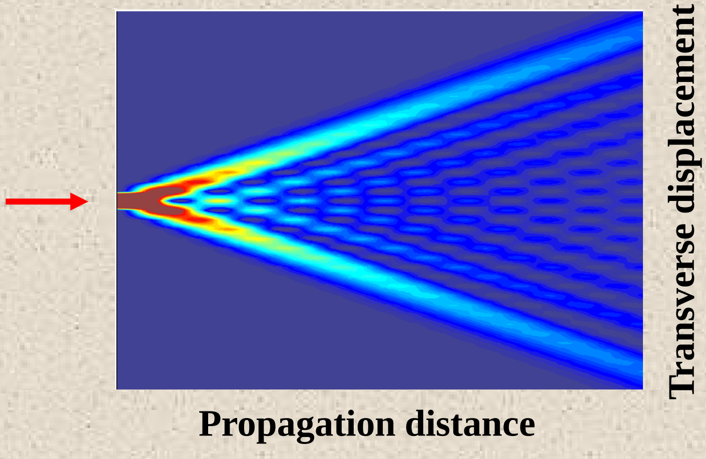
Christodoulides et al. (1988).

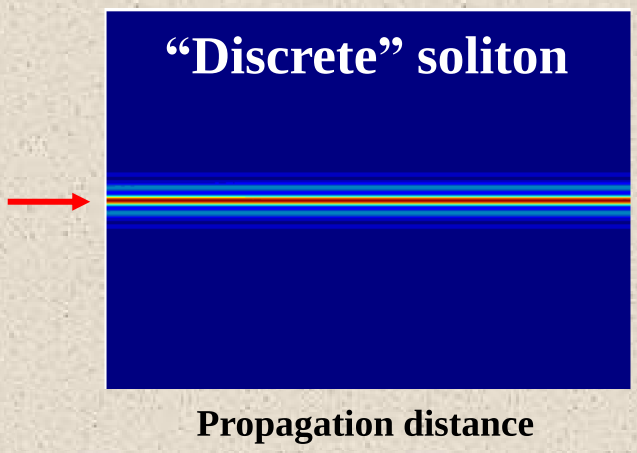
Eisenberg *et al.*, (1998).

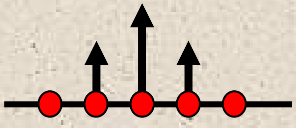
Lattice ("discrete") solitons:

When nonlinearity balances diffraction

Lattice (“discrete”) solitons



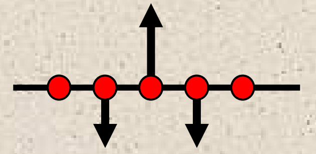




In-phase $k_x = 0$

D.N. Christodoulides et al. (1988).

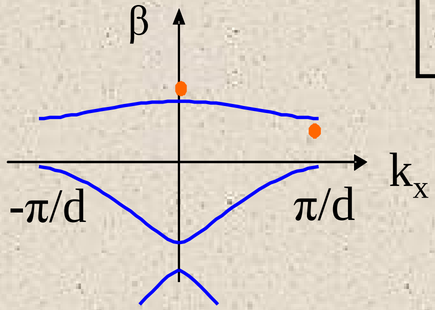
H.S. Eisenberg *et al.*, (1998).



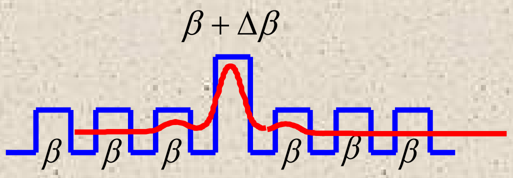
Gap (Staggered) $k_x = \pi/d$

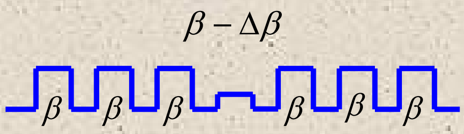
Y. Kivshar, (1993).

J.W. Fleischer *et al.*, (2003).



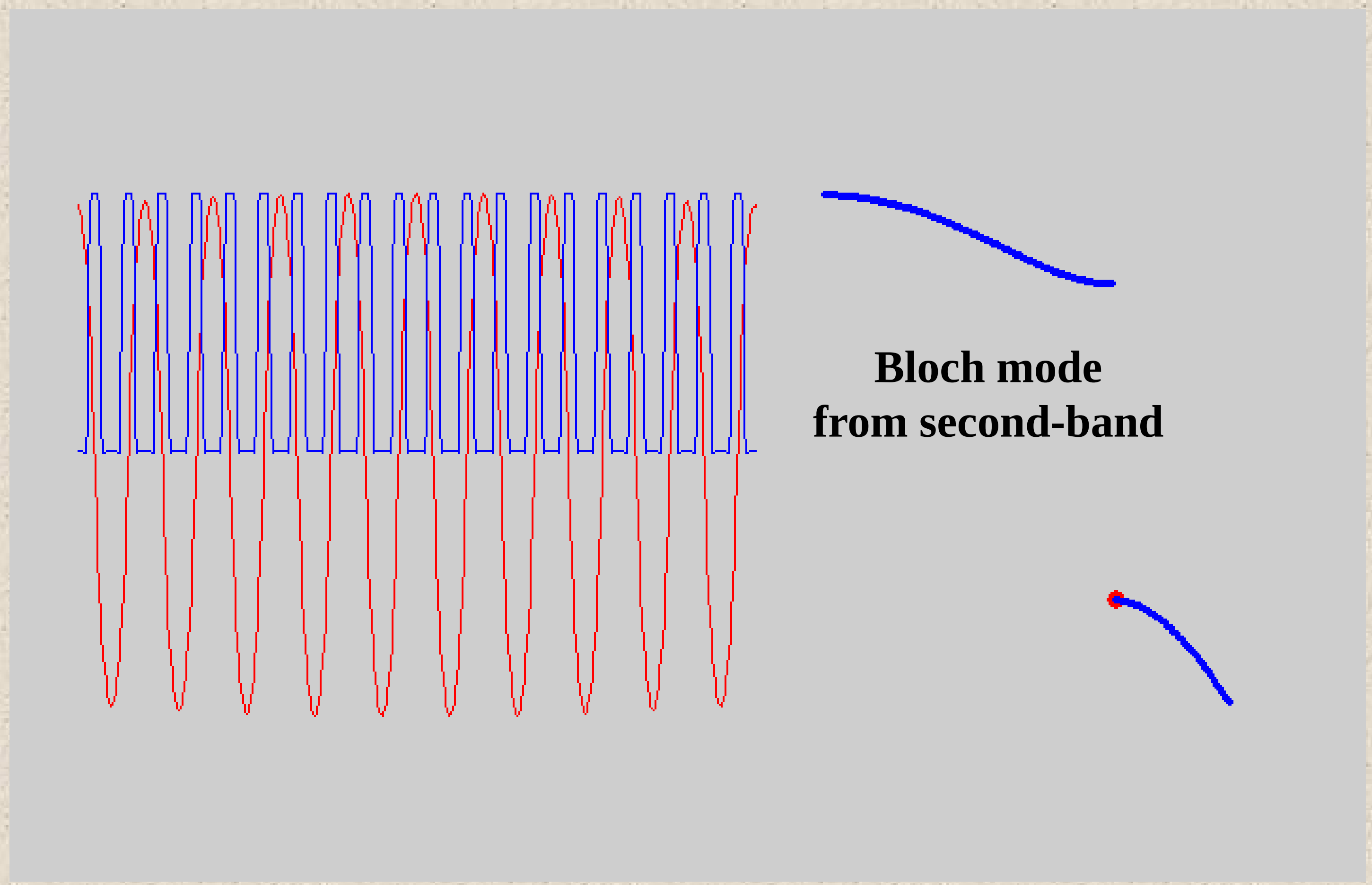
Lattice soliton is bound state of its own self-induced defect





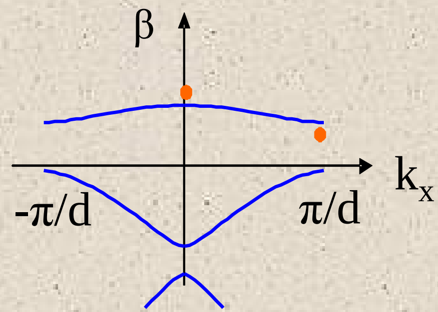
Example: 2nd-band lattice soliton

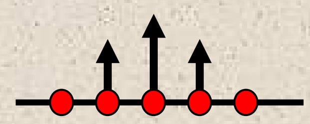
The soliton is a localized mode of the full potential (lattice + induced defect)



O. Cohen *et al*. PRL **91**, 113901 (2003).

Zoology of Lattice Solitons

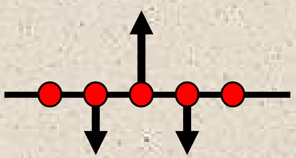




In-phase $k_x = 0$

D.N. Christodoulides and R.I. Joseph,
Opt. Lett. **13**, 794 (1988).

H.S. Eisenberg *et al.*
PRL **81**, 3383 (1998).



Gap (Staggered) $k_x = \pi/d$

Y.S. Kivshar,
Opt. Lett. **18**, 1147 (1993).

J.W. Fleischer *et al.*
PRL **90**, 23902 (2003)

2D: J.W. Fleischer *et al.*, Nature **422**, 147 (2003)

More recent:

- **Quadratic lattice solitons**
  Stegeman's group 2004

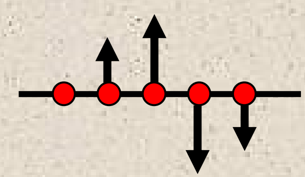
- **Lattice solitons in liquid crystals**
  Assanto's group 2004

- **Lattice modulation instability**
  Stegeman's group 2004

- **Collisions**
  Stegeman & Christodoulides' group 2005

- **Surface lattice solitons**
  Stegeman & Christodoulides group 2005

- **Zener tunneling**
  Krolikowski & Kivshar group

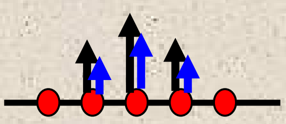


Twisted (dipole) $0 < k_x << \pi/d$

Darmanyan *et al.*,
Sov. Phys. JETP **86**, 682 (1998)

Neshev *et al.*, Opt. Lett. **9,** 710 (2003)

2D: Yang *et al.*, Opt. Lett. 29, 1662 (2004)



Vector at $k_x=0$

Darmanyan *et al.*,
Phys. Rev. E, **57**, 3520 (1998)

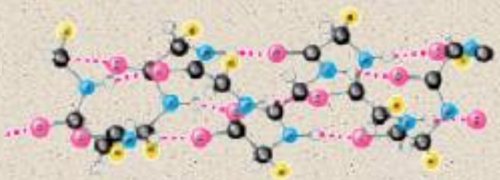
Meier *et al.*, PRL **91**, 143907 (2003).

2D: Chen et al., Opt. Lett. 29, 1656 (2004)

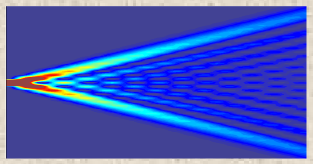
● Theory ● Experiment

Outline

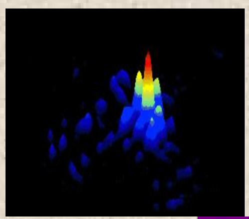
- Universality: nonlinear lattices in nature



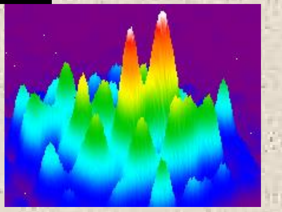
- Lattice transport and lattice solitons



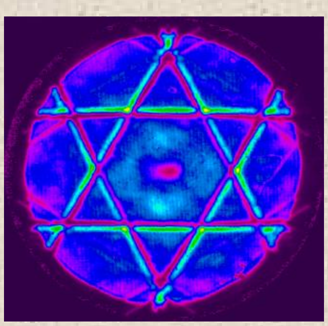
- 2D lattice solitons, vortices, etc.



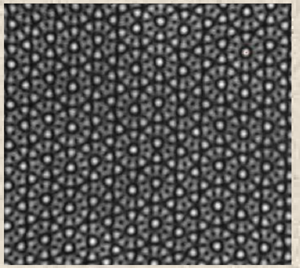
- Random-phase lattice solitons



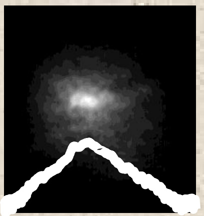
- Brillouin-zone spectroscopy of photonic lattices



- Nonlinear photonic quasi-crystals



- Anderson localization



State of the art in 2002: only 1D systems, only self-focusing

Transition to nonlinear 2D photonic lattices

But, how to make 2D nonlinear photonic lattices?

- **Direct fabrication:**

  - difficult
  - etching: practically 1D on the surface
  - fsec laser writing in glass: state of the art today 7x7 waveguides

- **Naturally-occurring 2D structure**

  - atomic scale ⍰ x-ray (linear + requires x-ray laser…)
  - none found so far

All-optical induction !

à la holography

Using Nonlinear Optics to Induce Nonlinear Waveguide Arrays

Requirements

- **Lattice waves propagate *linearly***
- **Probe wave feels periodic potential AND nonlinear self-focusing**

➔To do that, we need a strong nonlinear optical anisotropy

e.g. photorefractive (SBN-75) crystal

Create WGs by interfering pairs of plane waves

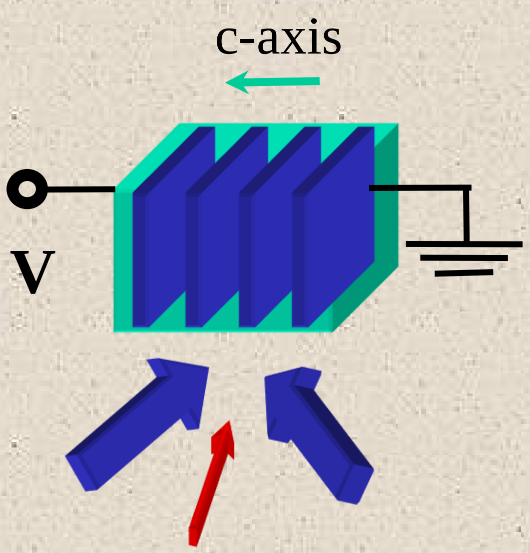
– polarize ⊥ to c-axis

$I_{grating}$:I_{probe} ~ 5-10:1

Polarize probe || to c-axis

Apply Voltage || to c-axis

- Nonlinearity: $\Delta n \sim 10^{-3}$

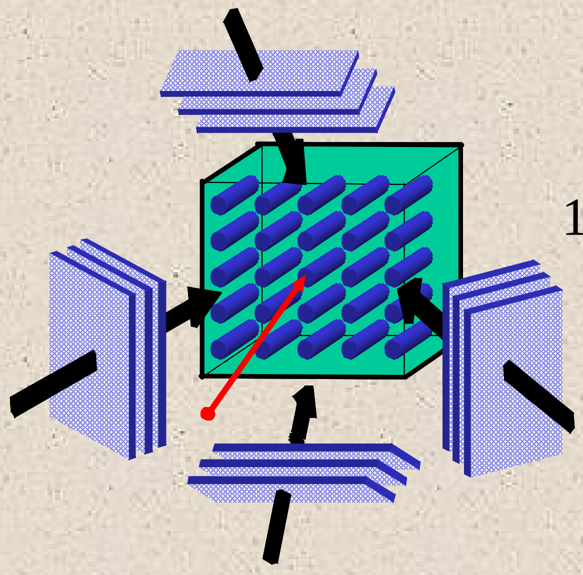


V > 0 ⇒ focus,
V < 0 ⇒ defocus

- **can induce any lattice we want**
- **can have either self-focusing or defocusing nonlinearities**

Optical Induction of 2D Array of 2D Waveguides

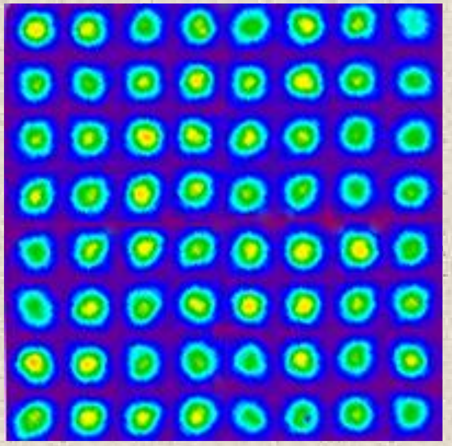
Interfere 2 pairs of plane waves to create dynamic 2D array



15mW in each plane wave

$I_{grating}:I_{probe} = 5:1$

Crystal output

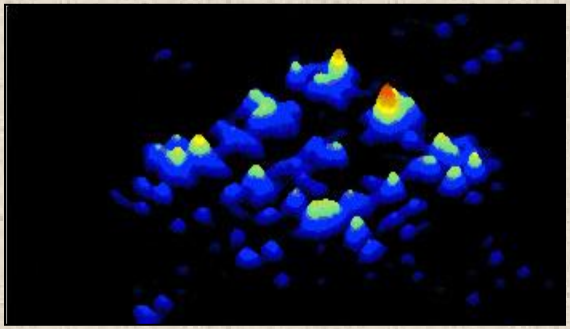


Representative square array
WG diam = 7μm D=11μm

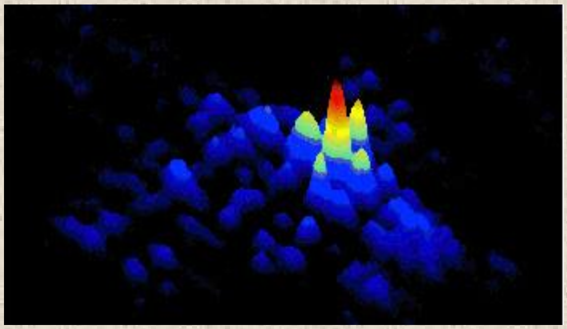
Strong anisotropy of crystal allows distinction between induced lattice (waveguide array) and probe beam

First 2D Lattice Soliton in any system in nature

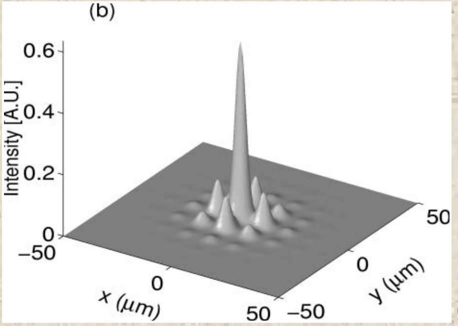
Lattice diffraction



2D Lattice Soliton
(high intensity + self-foc)

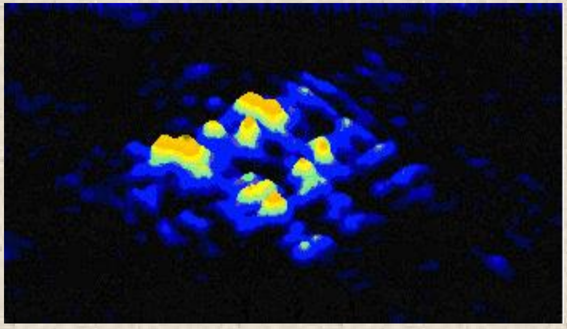


Soliton simulation

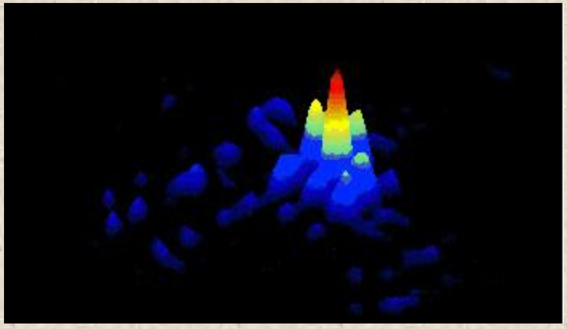


8x reduction of I_{probe}

Interference
with plane waves



Low intensity gives
back linear diffraction



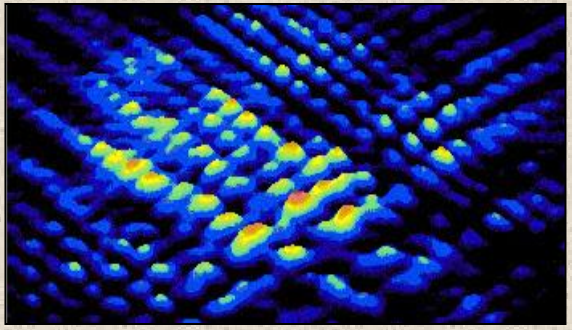
Interferogram

(800V)

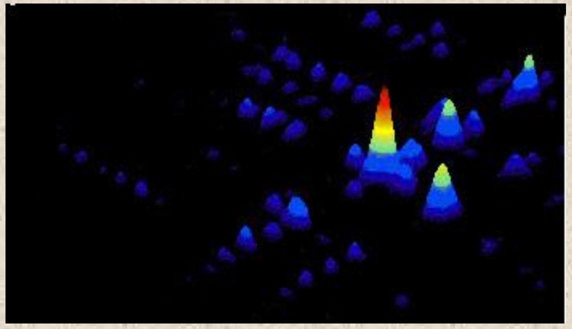
J.W. Fleischer *et al.*,
Nature **422**, 147 (2003)

First 1D Gap Soliton

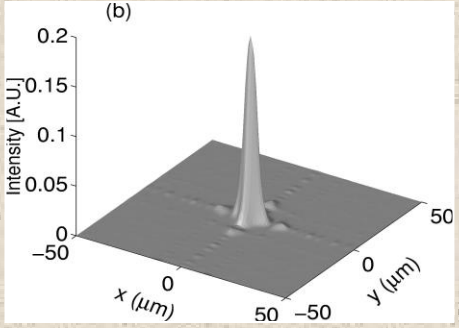
Lattice Diffraction



2D Gap Lattice Soliton
(high intensity + self-defoc)



Soliton simulation



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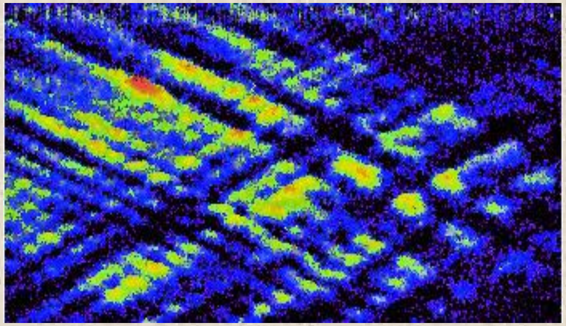

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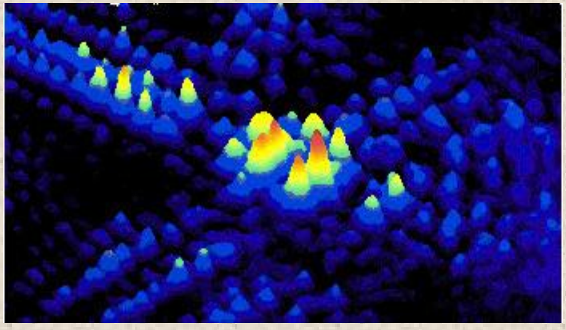

Interferogram

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Vortex-Ring Lattice Solitons

First lattice solitons with angular momentum

- ***in general,***
  angular momentum in a lattice not conserved

But

"on axis", vortex lattice solitons do exist!

Prediction of vortex lattice solitons:

- Malomed & Keverkidis, PRE 2001
- Yang & Musslimani, Opt. Lett. 2003

Experiments:

Fleischer *et al.*,
PRL **92**, 123904 (2004).

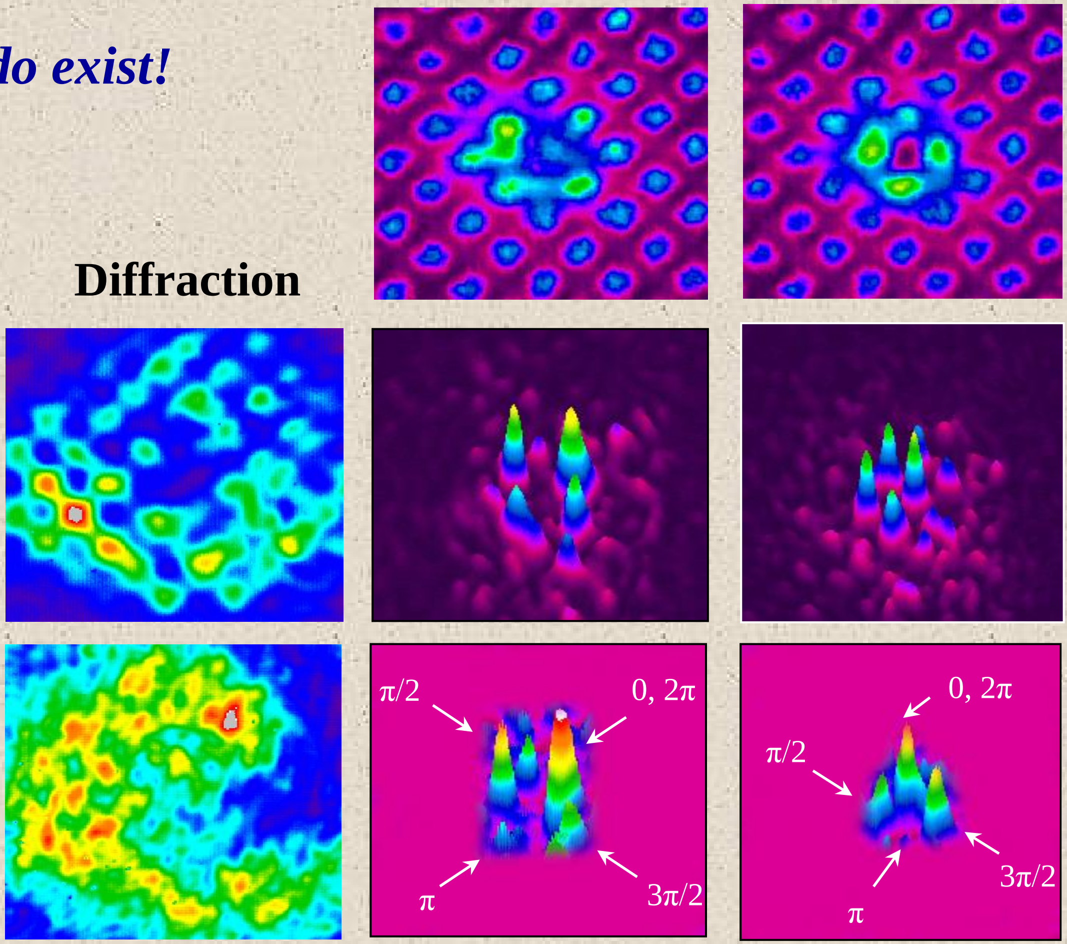
Neshev *et al.*
PRL **92**, 123903 (2004).

On-site **Off-site**

Diffraction

Intensity

Phase



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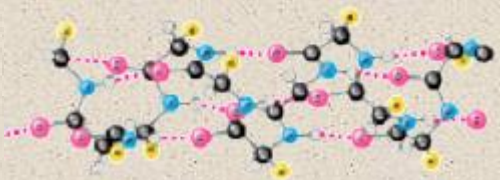

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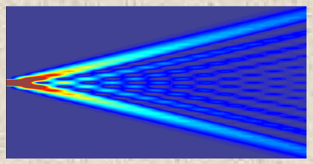

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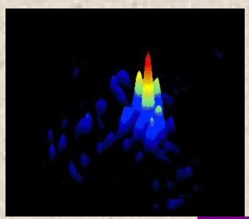

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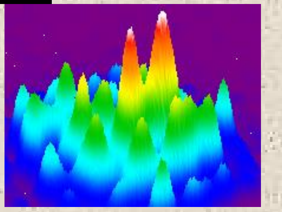

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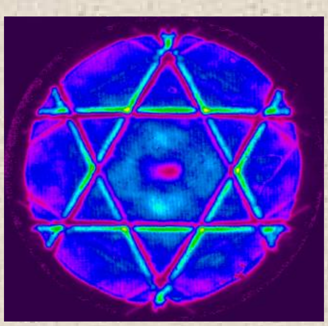

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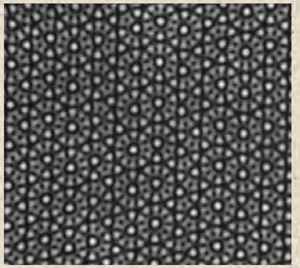

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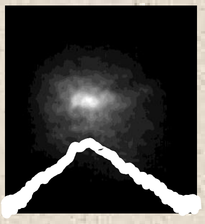
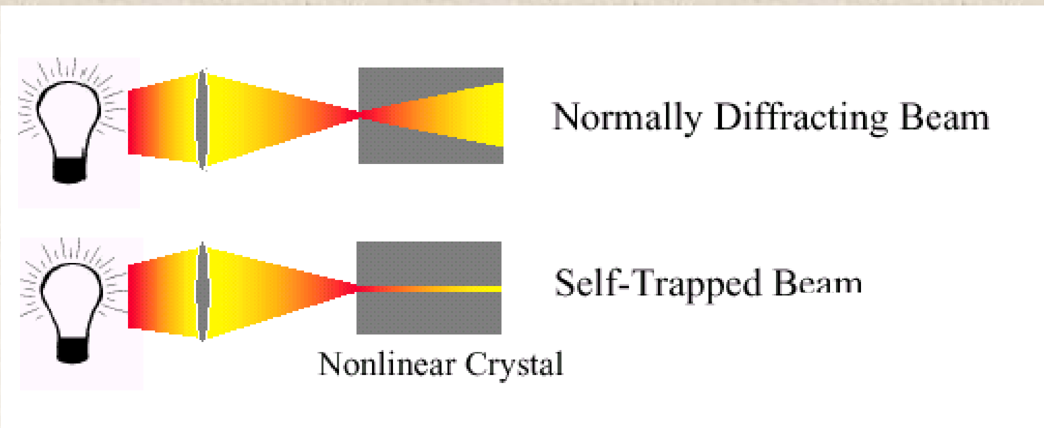
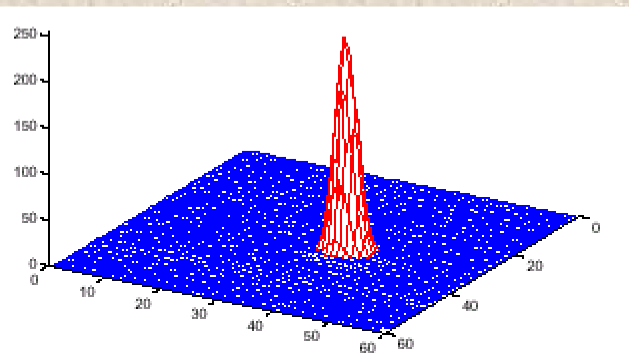


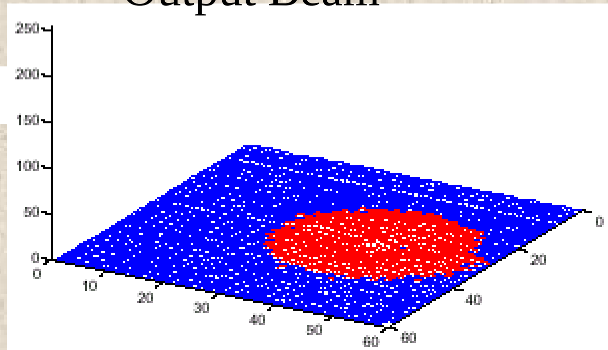
Incoherent (Random-Phase) Solitons:
Solitons made of white light!



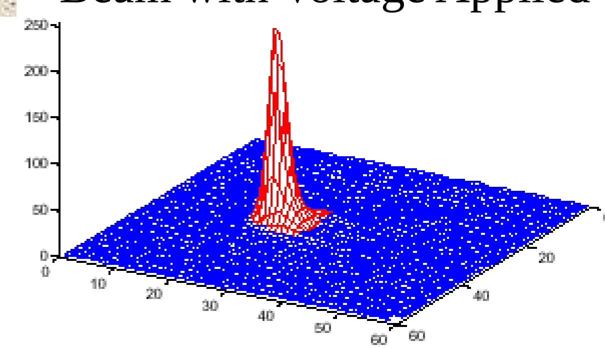
14 μm Input Beam



82 μm Diffracted Output Beam



12 μm Self-Trapped Output Beam with Voltage Applied



Incoherent beams can be trapped in any non-instantaneous nonlinearity.

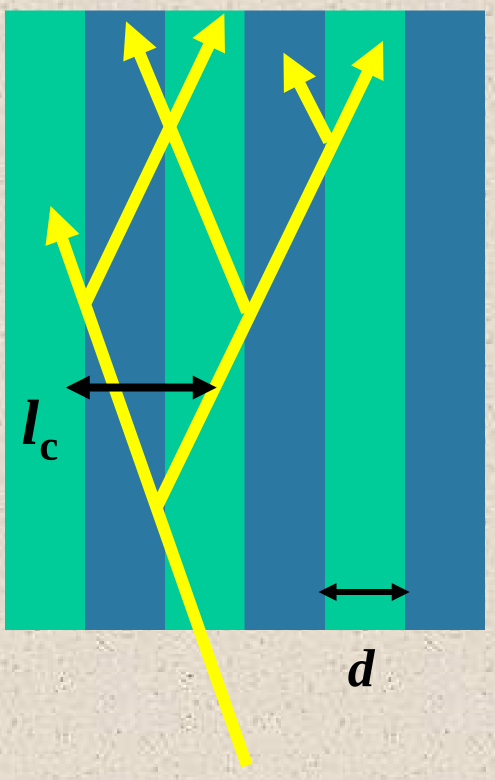
Incoherent Bright Solitons - M. Mitchell and M. Segev, Nature, 387 (1997)

Incoherent Dark Solitons - Z. Chen, M. Mitchell, M. Segev, T. Coskun, D. Christodoulides, Science, 280 (1998)

Random-Phase Lattice Solitons

All **previous research on lattice solitons performed with *coherent waves***

Phase is perfectly correlated in space & time (e.g., staggered, vortex, …)



Propagating waves undergo multiple reflections

- **Interference phenomena drive dynamics**
  - **greatly affected by coherence of waves**
- **Explore with statistical (spatially-incoherent) light**

Correlation length l_c vs. lattice spacing d

Random nature of light vs. periodic constraint

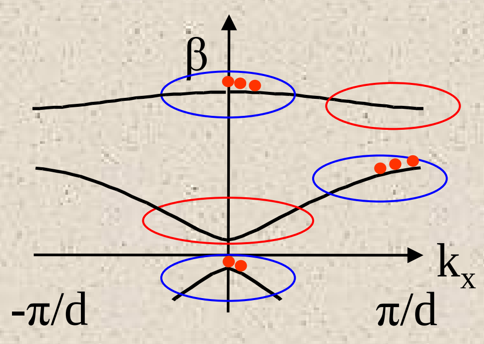
$$\left\langle \psi(x_1,z,t)\psi^*(x_2,z,t)\right\rangle_\tau \qquad f(x)=f(x+d)$$

(Mutual correlation) **(Bloch's theorem)**

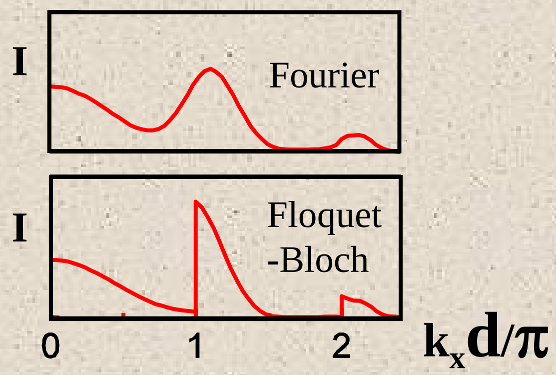
Have lattice soliton with randomly-varying phase front

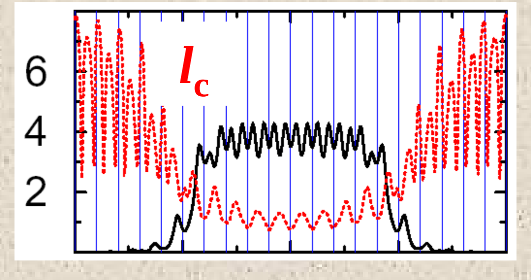
Prediction of Random-Phase Lattice Solitons

Transmission spectrum



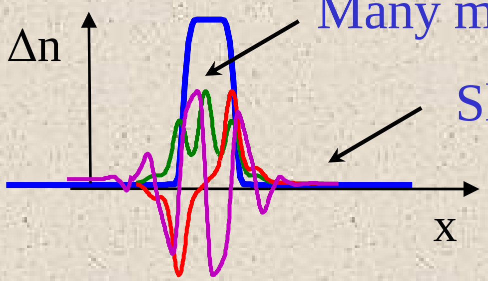
⟨**Power spectrum**⟩





Note $l_s(x) = l_s(x+mD)$

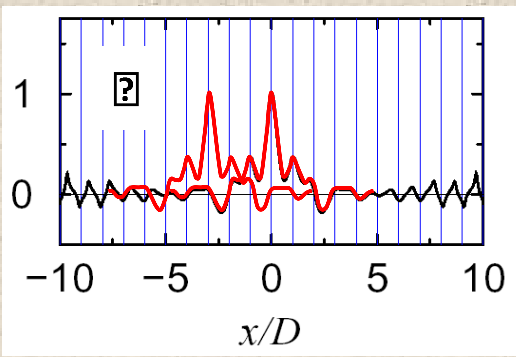
Interpretation using modal theory



Many modes → low l_c

Slowest-decaying mode

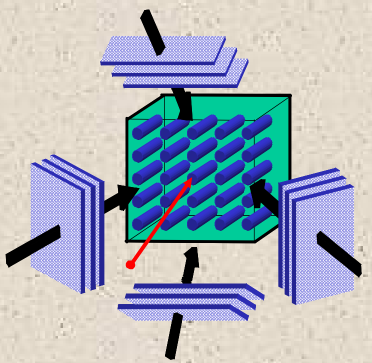
→ higher l_c

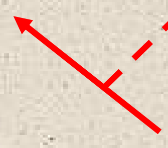


Mutual coherence/correlation function $\mu = \frac{\langle \psi_n(x)\psi_n(x+\delta)\rangle_X}{\sqrt{I(x)I(x+\delta)}}$

H. Buljan *et al.*, Physical Review Letters 92, 223901 (2004).

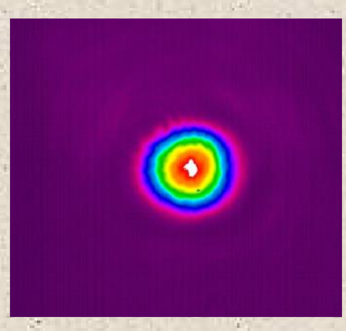
Spatially-incoherent wave-packets





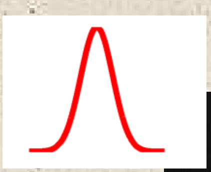
Array beams define Bragg angles

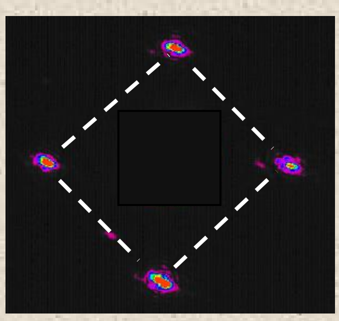
Input beam



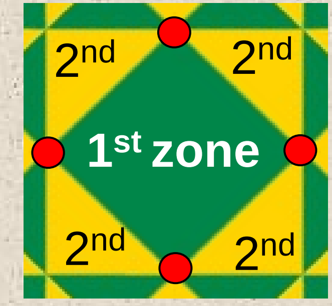
No diffuser (coherent)

With diffuser (incoherent)



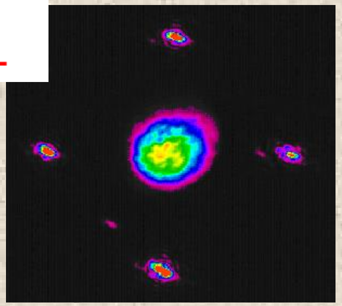


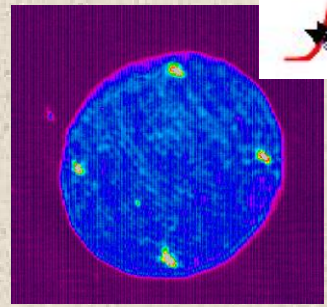
Points from array



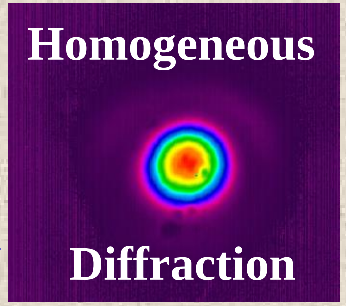
Brillouin zones

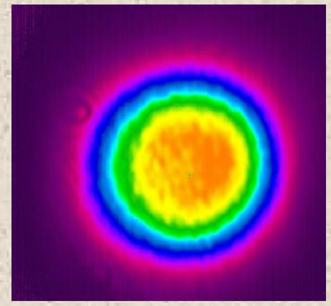
Fourier space





Real Space output





coherent

incoherent

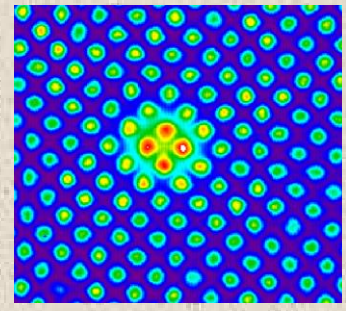
Diffraction in homogeneous medium

determined by λ/L $\left(\lambda/w \text{ vs. } \lambda/l_c\right)$

Random-phase lattice solitons – Experimental results

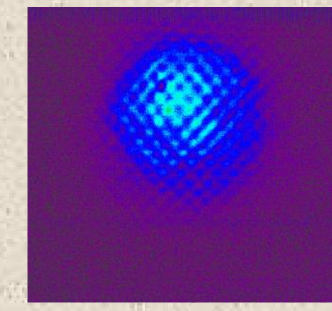
Real space

Input

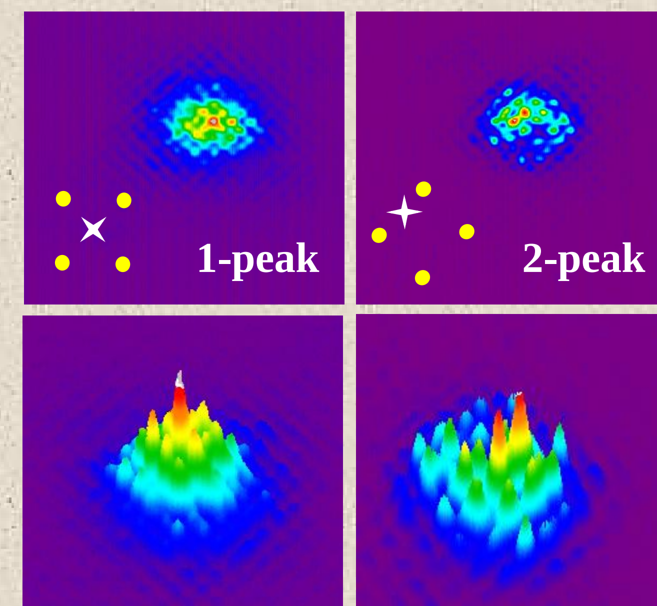


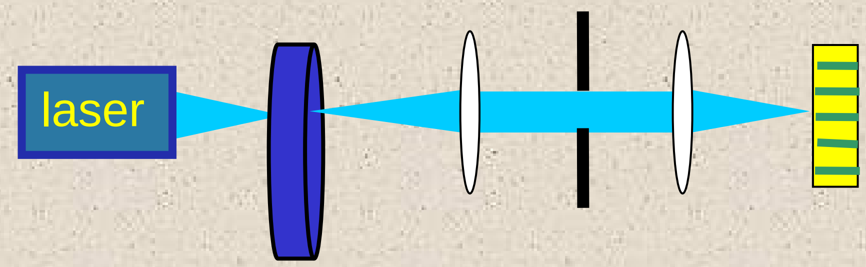
d = 11.5 microns

Diffraction



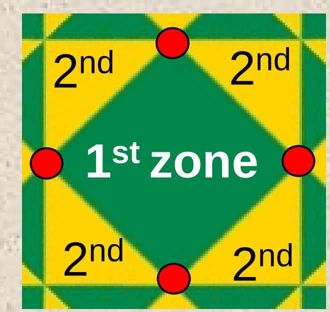
Random-phase lattice soliton





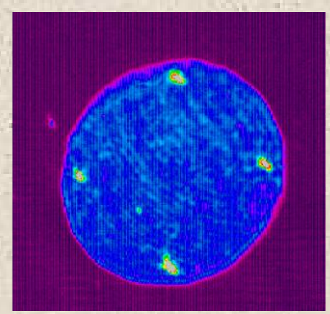
Rotating diffuser

correlation length $l_c < d$

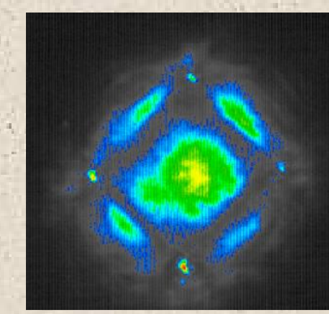


Brillouin zones

***Brillouin zone evolution*!!**



Incoherent input beam



Soliton output

O. Cohen *et al*. Nature 433, 500 (2005).

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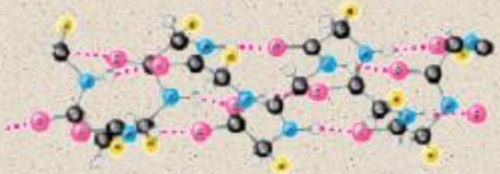

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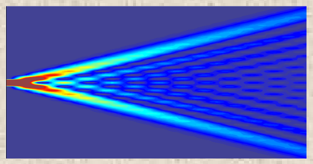

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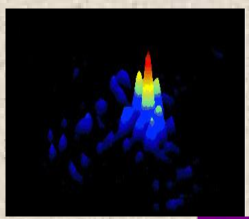

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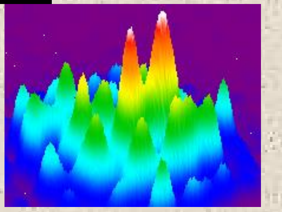

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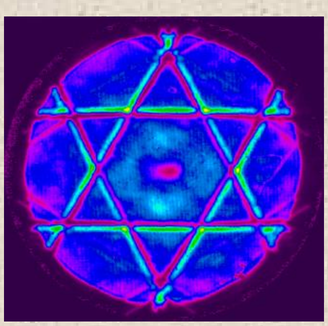

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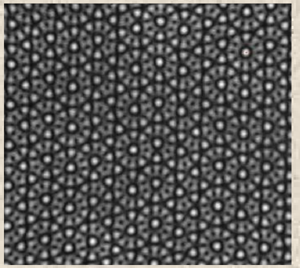

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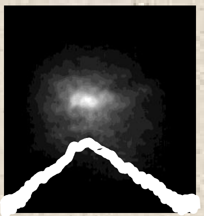
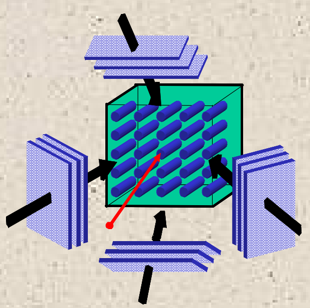
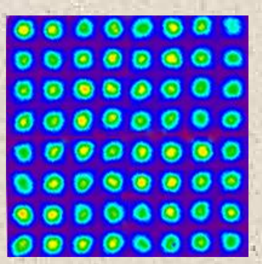
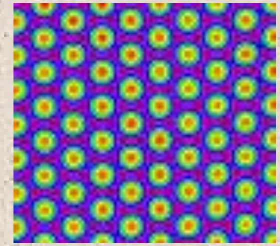


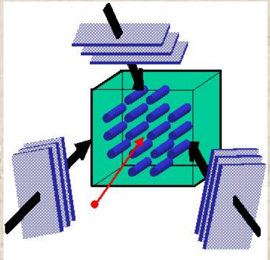
Brillouin-zone spectroscopy of photonic lattices

Experimental lattice



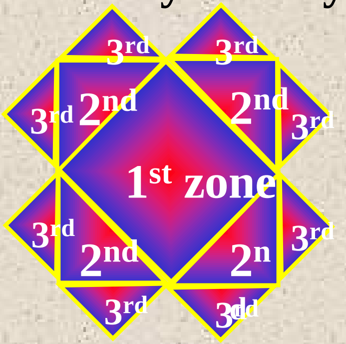


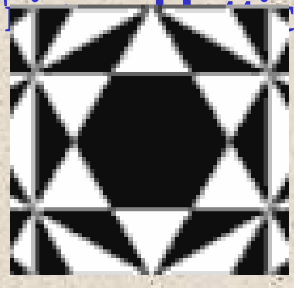




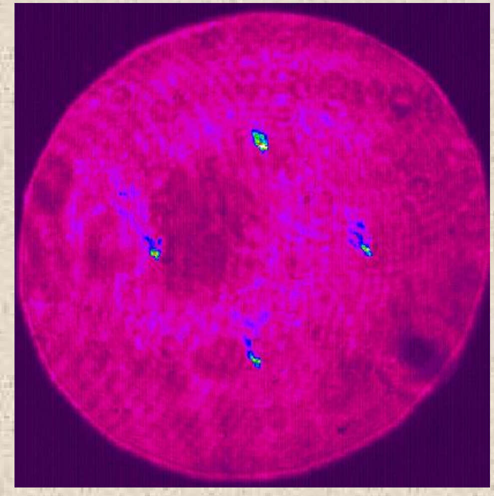
4-fold symmetry **3-fold symmetry**

Theoretical Brillouin zones





Spatially-incoherent probe beam



wide in both real space and Fourier space

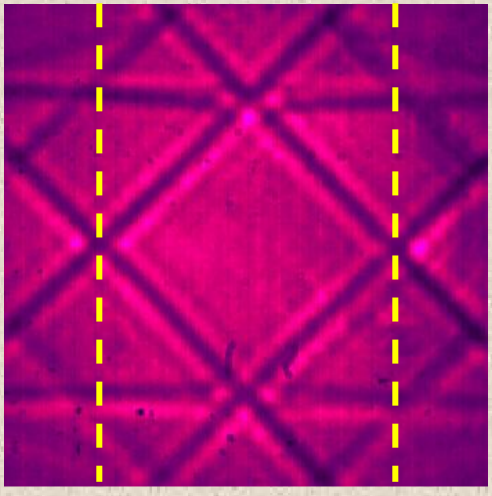
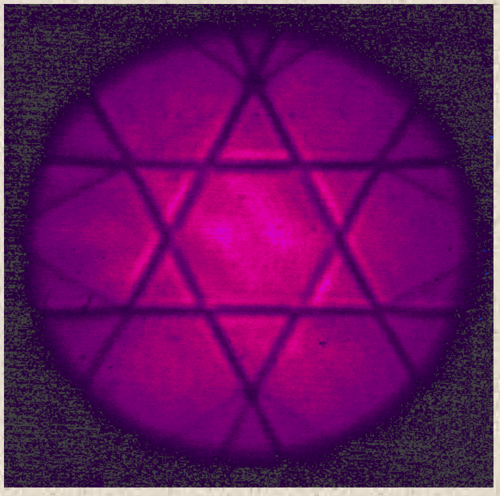


Image of output



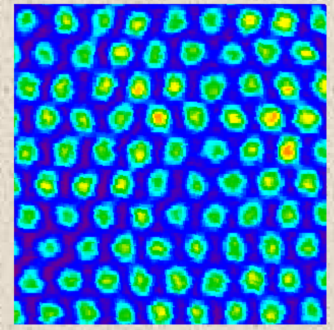
Made in Israel

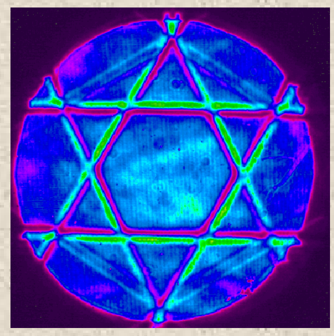
Photonic lattices with defects

Our experiments

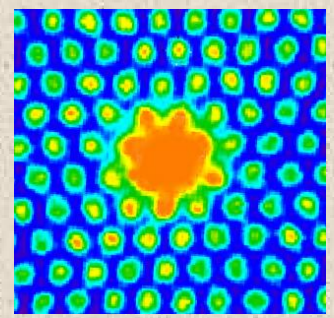
Real space | **K-space**

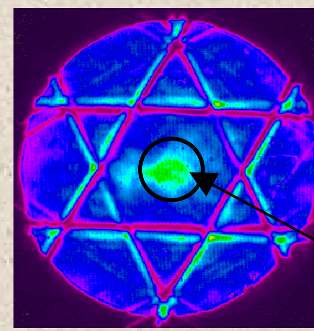
No defect





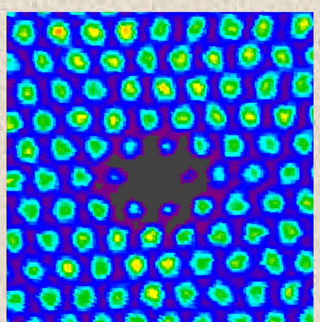
Positive defect

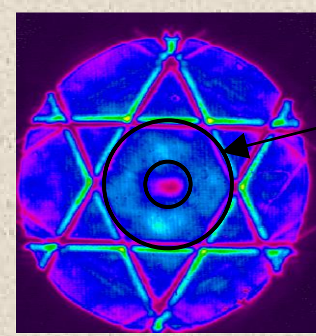




Guided Modes

Negative defect

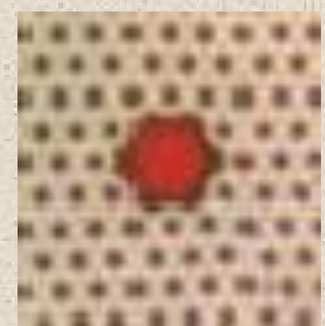


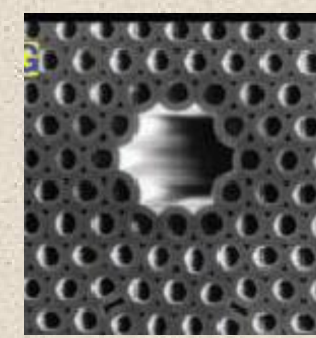


Photonic Crystal Fibers

P.S. Russell, Science **299**, 258 (2003).

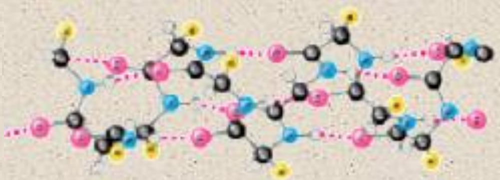
Far-field dispersion | **Real space**



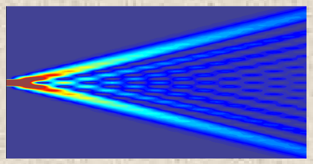


Outline

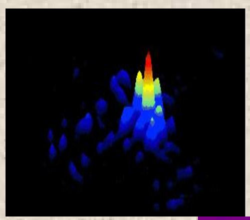
- Universality: nonlinear lattices in nature



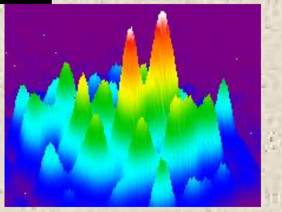
- Lattice transport and 1D lattice solitons



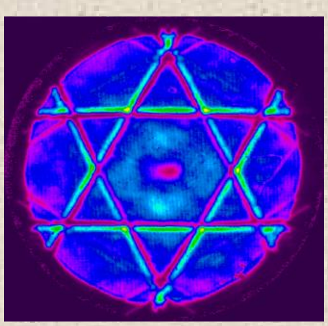
- 2D lattice solitons, vortices, etc.



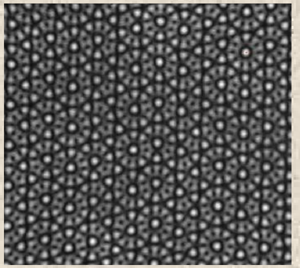
- Random-phase lattice solitons



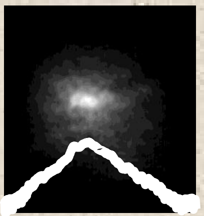
- Brillouin-zone spectroscopy of photonic lattices



- Nonlinear photonic quasi-crystals



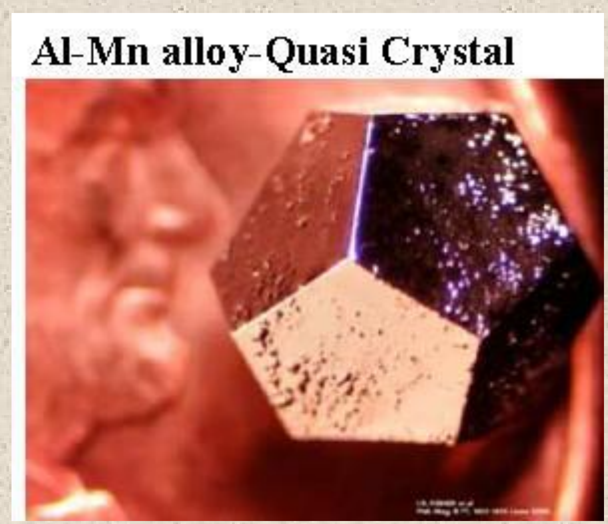
- Anderson localization



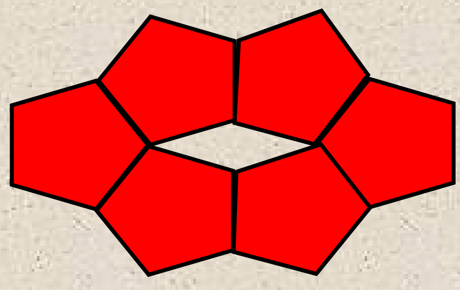
Nonlinear Photonic Quasi-Crystals

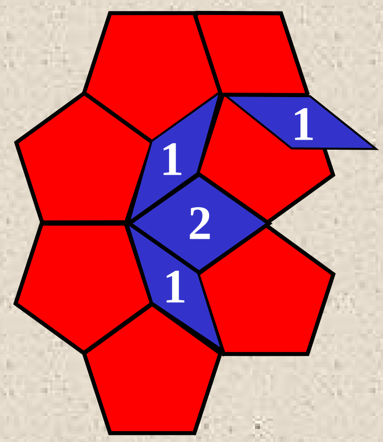
- What are quasicrystals?
  - Classical crystal definition: *periodic structure*
  - Rotational symmetry of orders 2,3,4,6 only.
  - 1982: QC discovery by Shechtman; crystal redefined.

  - Non-periodic
    - Long range order
    - No invariance under space translations or rotations
    - No "unit cell"

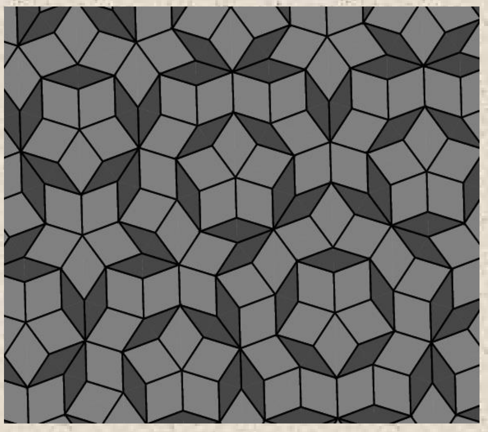


Lattice Symmetry





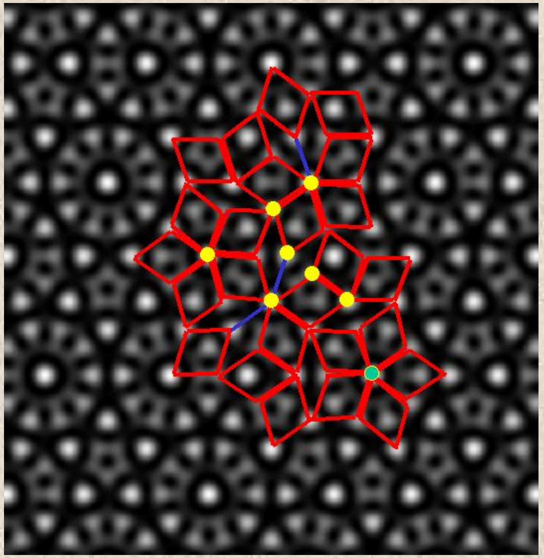
Overlapping pentagons



Quasiperiodic pattern

- Can be tiled using "**Penrose Tiling**".
- More than one possible tiling for a given quasicrystal structure.
- True for rotational symmetry of order 5, 7, 8, 9, etc. (no upper bound)

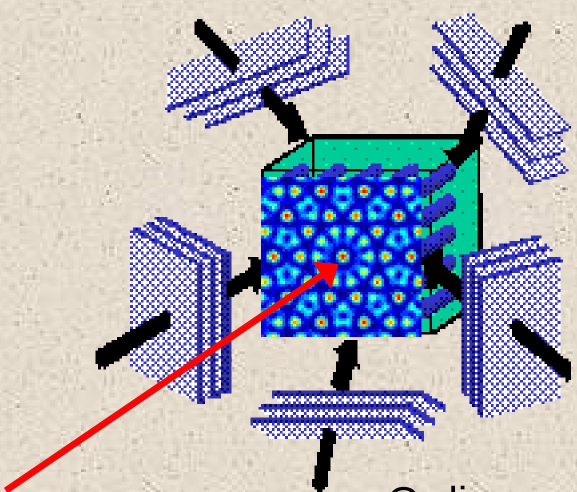
2



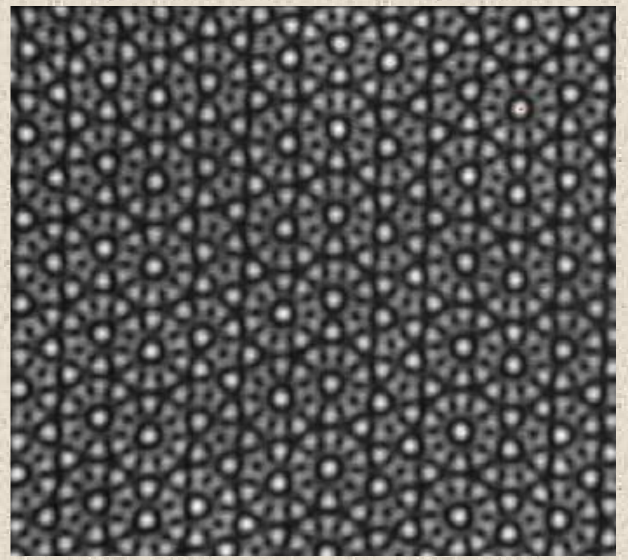
Making a Nonlinear Photonic Quasi-Crystals

Use optical Induction to make the quasi-crystal:

Interference of 5 waves



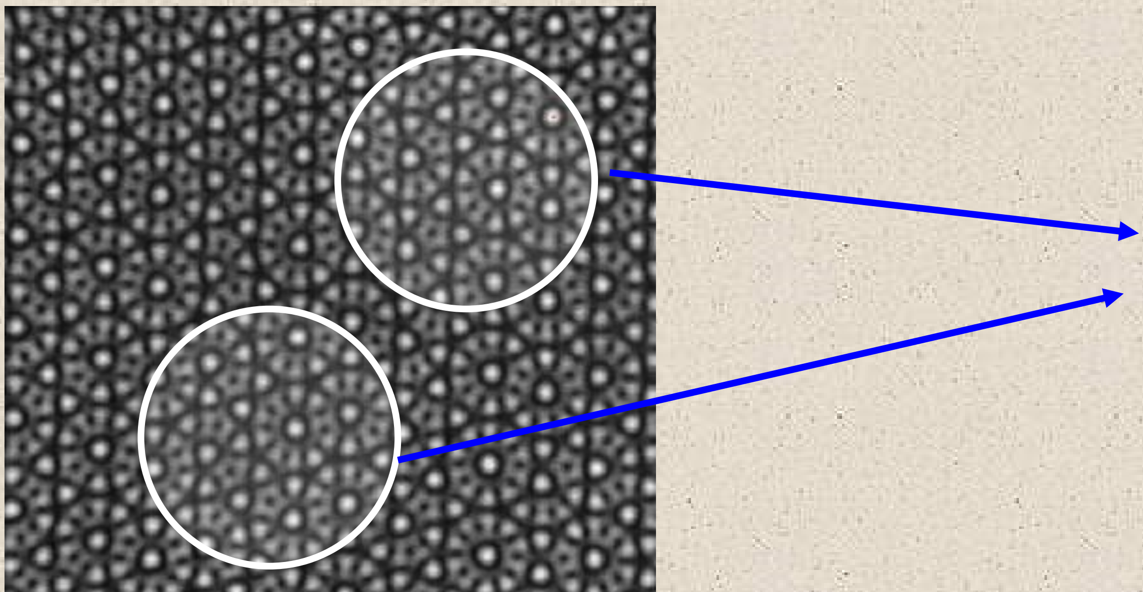
5-fold optically induced Quasi-Crystal



Experimental picture

Freedman et al. Nature, **440**, 1166-1169 (2006).

How do you know it's a crystal?

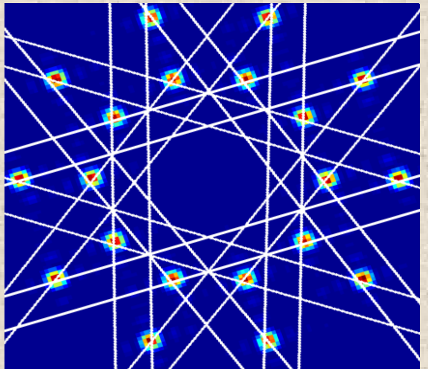


These don't look the same…

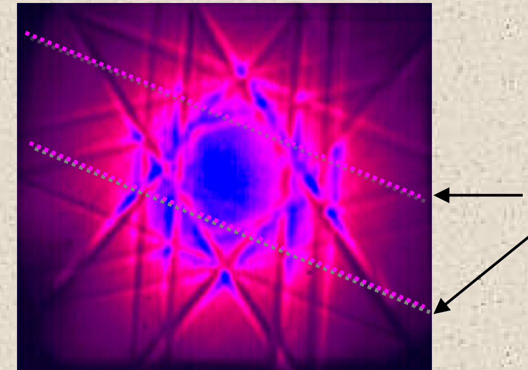
Or do they?

- Look for *statistical* similarity of different regions by looking at the Fourier space
- Examine Bragg diffraction pattern

Quasi Brillouin Zones



Theory

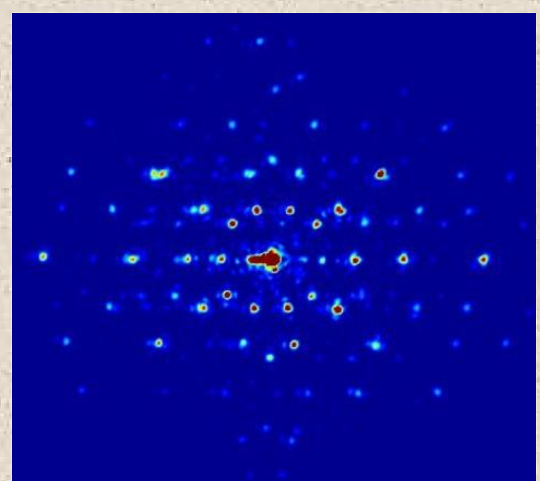


Lines appearing "weak" in the experiment

Experiment

- pictures of the quasi Brillouin zones at different spots of the crystal
- Same scattering at different spots → Statistical similarity → It's a crystal.

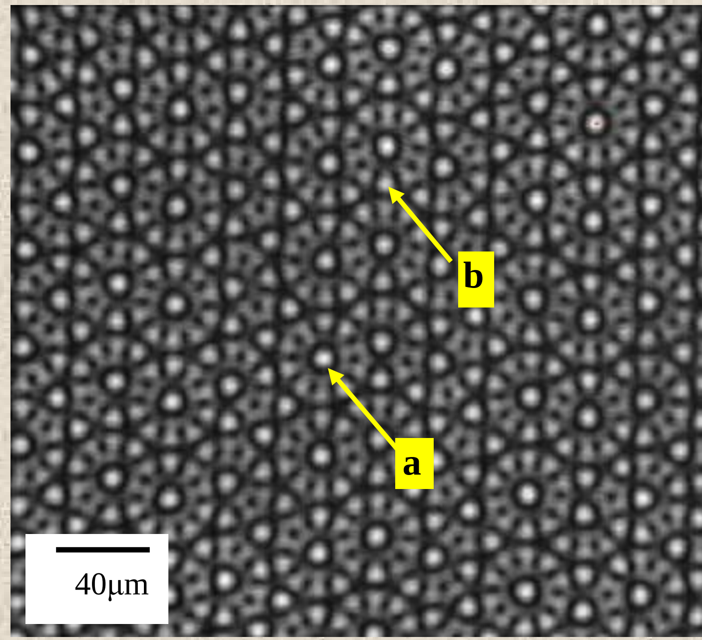
Bragg Diffraction Pattern



- Just like crystallography. (TEM image).
- Testing for the "Real" photonic structure by looking at the diffraction pattern.
- "Higher Order" Bragg peaks prove the similarity to atomic quasicrystals.

Tunneling / Discrete Diffraction

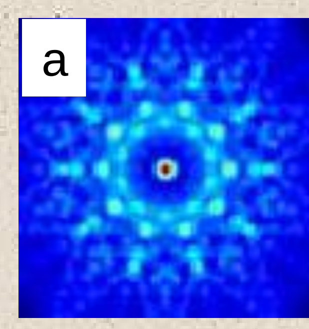
5 fold optically induced Quasi-Crystal

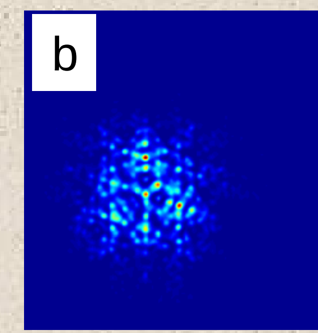


Experimental picture

Output after 5mm propagation

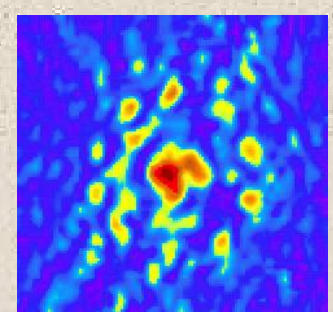
Theory

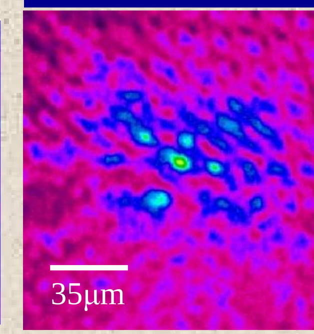




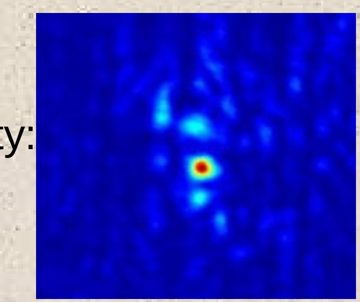
Experiment

Low intensity: Diffraction





High intensity: **Soliton**

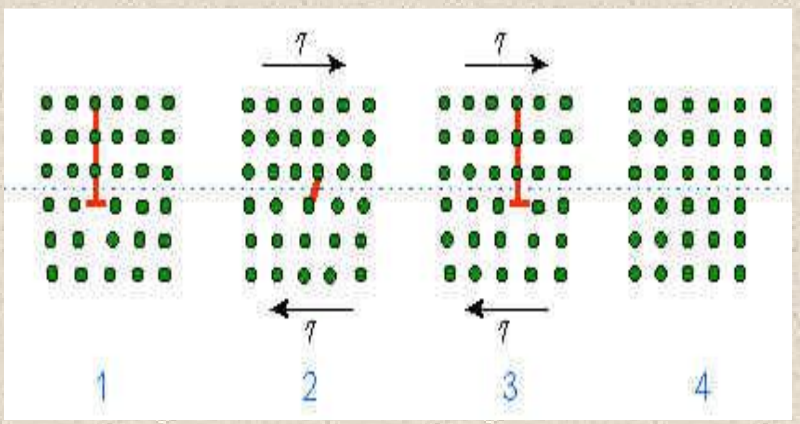


Freedman et al. Nature, **440**, 1166-1169 (2006).

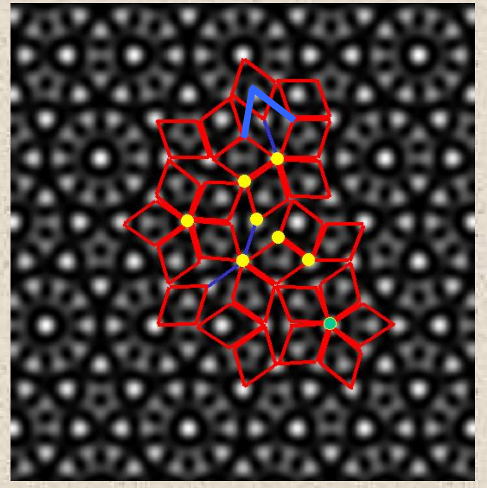
Defect dynamics

Interacting lattices.

- Dislocation in a ***square lattice*** → moves by "jumping" from one row of atoms to the next, without leaving traces of its movement.
- Dislocation in a ***quasicrystal*** → leaves a "cloud" of misplaced atoms behind it, with a finite relaxation time.
- nontrivial readjustment of the sites - **phason flip**.



Movement of a line dislocation in a square lattice

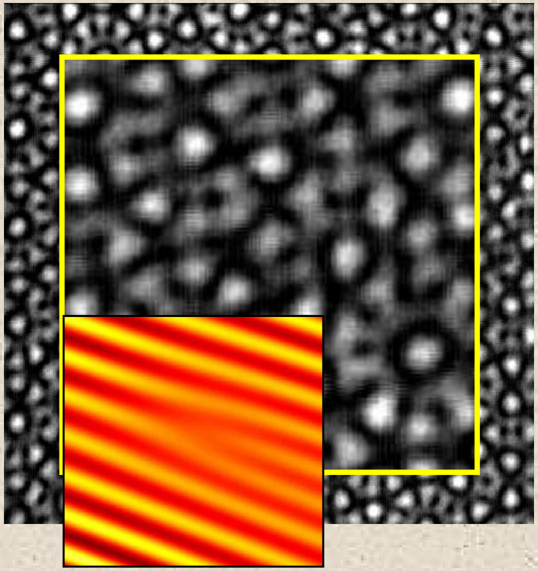


Blue lines indicate possible “Phason Jumps”

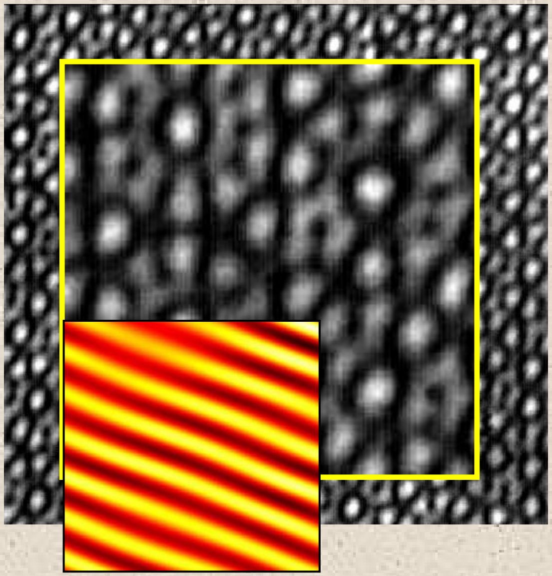
Experiments with QC lattice defects

Interacting (nonlinear) quasi-lattice: defect relaxation

Non-interacting lattice, with a dislocation



Lattice rearranged - no dislocation



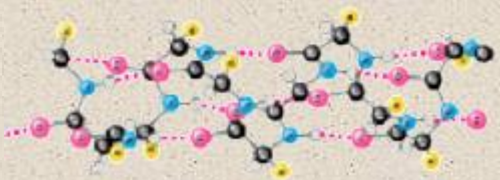
Nonlinear interacting lattice - Long range rearrangement

Output face pictures

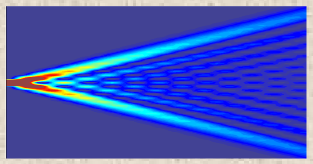
Freedman et al. Nature, **440**, 1166-1169 (2006).

Outline

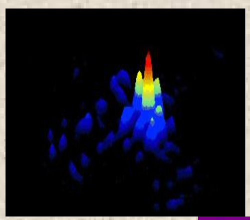
- Universality: nonlinear lattices in nature



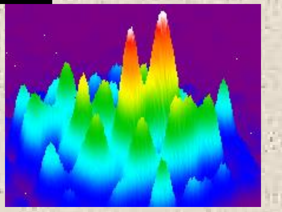
- Lattice transport and 1D lattice solitons



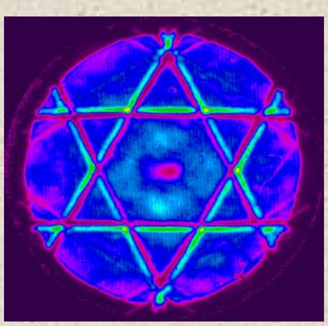
- 2D lattice solitons, vortices, etc.



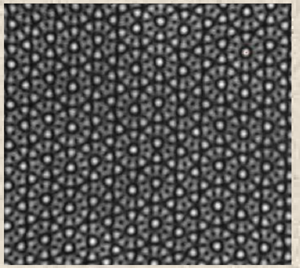
- Random-phase lattice solitons



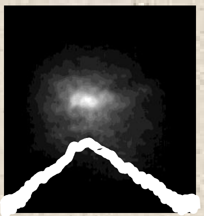
- Brillouin-zone spectroscopy of photonic lattices



- Nonlinear photonic quasi-crystals



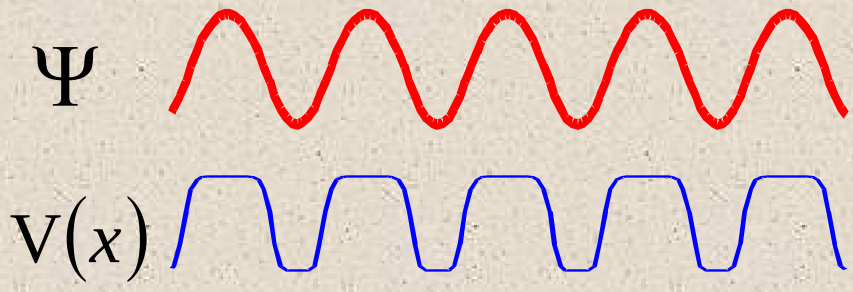
- Anderson localization



Anderson Localization in Perturbed Photonic Lattices

Periodic Lattice:

Bloch waves
(extended states)

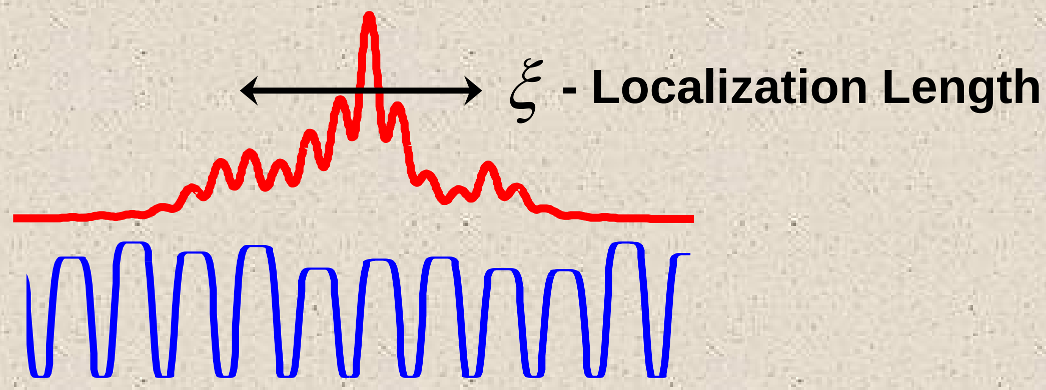


A wave propagates freely through the medium
Ballistic Transport/Diffraction

Disordered Lattice:

Localized States

Typical scale ξ



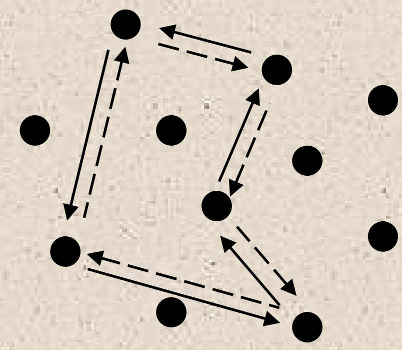
The wave can remain confined in some region of the potential

Philip W. Anderson, 1958 (Nobel Prize 1977)

Localization:
arising from multiple coherent scattering from defects

Scattering by impurities generates random walk

The wave is **coherently** scattered by defects



➡ Constructive interference between recurrent scatterings

➡ Higher return probability ➡ Quantum corrections to diffusion, Localization

From ballistic transport to diffusion to localization

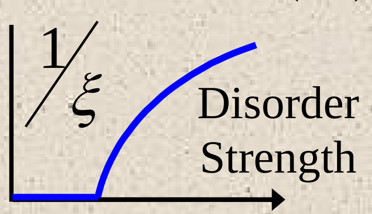
Dimensionality:

1-D $\xi \sim l$ ← *mean free path*

2-D $\xi \sim l \exp(kl)$

} All modes are localized, always

3-D



Localization above critical disorder,

Phase Transition (Metal-Insulator)

Localization of Light – Brief Overview

Localization effects originally suggested for electrons

General for all wave systems!

EXPERIMENT	Albada & Lagendijk, PRL 1985 Wolf & Maret, PRL 1985 Etemad & Andrejco, PRL 1986	Weak localization of photons: Coherent Back-Scattering
THEORY	Akkermans *et al.*, PRL 1986	
THEORY	S. John, PRL 1984	Strong localization in super-lattices
EXPERIMENT	Wiersma *et al.*, Nature 1997 Chabanov *et al.*, Nature 2000	Strong localization effects in transmission through 3-D suspensions

EXPERIMENT

Pertsch *et al.*, PRL 2004 — 2D dynamics in disordered waveguide arrays

Fixed randomization of sites location due to fabrication process

Anderson Localization in periodic structure never observed!

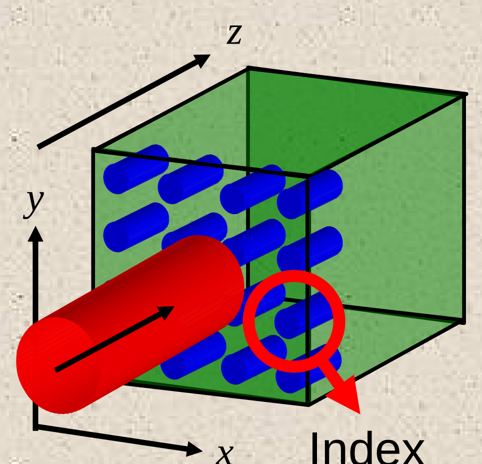
(2+1)D Dynamics in 2D Photonic Lattices

Objective:

Anderson Localization in 2D photonic lattices:

Dynamics (i.e. diffraction) in transverse plane

Propagation direction replaces time



Index modulation $\sim 10^{-4}$

$$i\frac{\partial \Psi}{\partial z} + \frac{1}{2k}\nabla_{\perp}^{2}\Psi + \frac{k}{2n_0}\Delta n(x,y)\Psi = 0$$

Photonic Lattice

Schroedinger-like Equation

Similar to

2D electron system

2D BEC

Transverse localization of light suggested by Lagendijk 1989

Expected Dynamics:

At a finite distance:

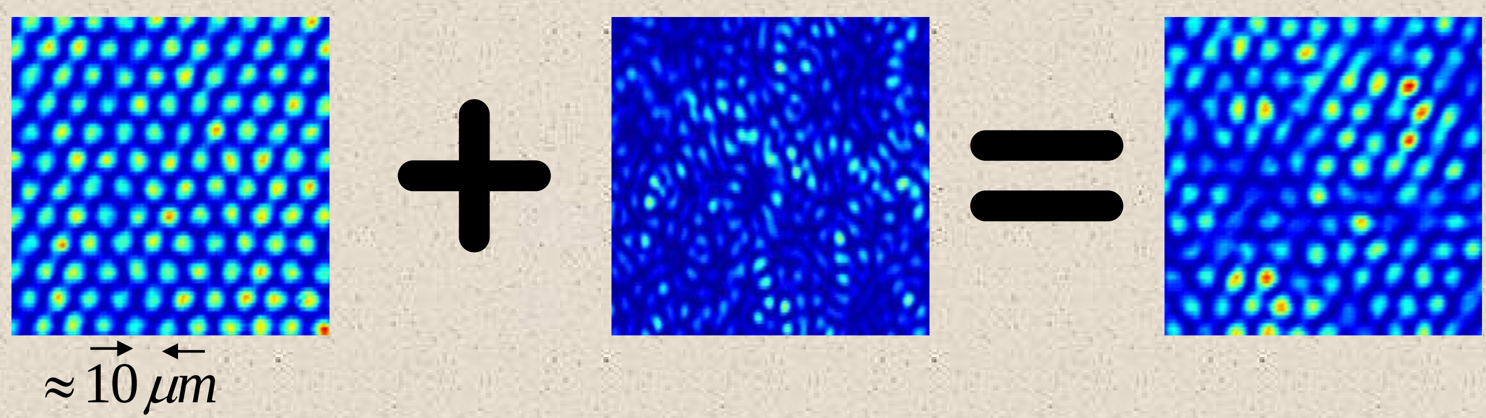
Ballistic Transport → Diffusive Regime → Localization

Level of disorder →

Making a perturbed lattice
using the optical induction technique:

Interfere 3 beams on a photo-sensitive anisotropic material (SBN:75)	Add another beam, passed through a diffuser (= random speckles)

Hexagonal Lattice | **Controlled Level of Disorder**



- Rotate the diffuser – different realization, same statistics

Similar ideas recently suggested (but never demonstrated) for matter waves (BEC):	U. Gavish et al., J.E Lye et al., R.C Kuhn *et al.* all in PRL 2005

Problem: Making a propagation-invariant disordered lattice

Anderson localization is destroyed if the potential varies with propagation

We use a diffuser for generating disorder by a speckled intensity pattern

Narrow speckles diffract

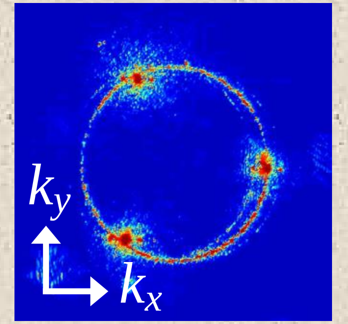
Non-stationary propagation of writing beams

The lattice changes along propagation direction

(= time-dependent potential)

Localization effects are destroyed!

Solution: random superposition of Bessel beams



Lattice Beams Far-Field

$$k_z^{\;2} = k - \left(k_x^{\;2} + k_y^{\;2}\right)$$

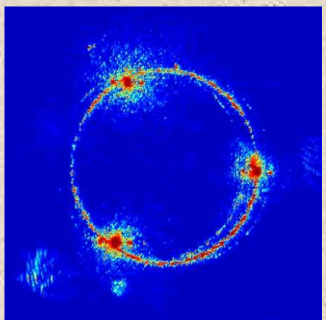
All Fourier components travel with the same velocity

Z-Independent Perturbed Lattice

Making A Disordered Lattice

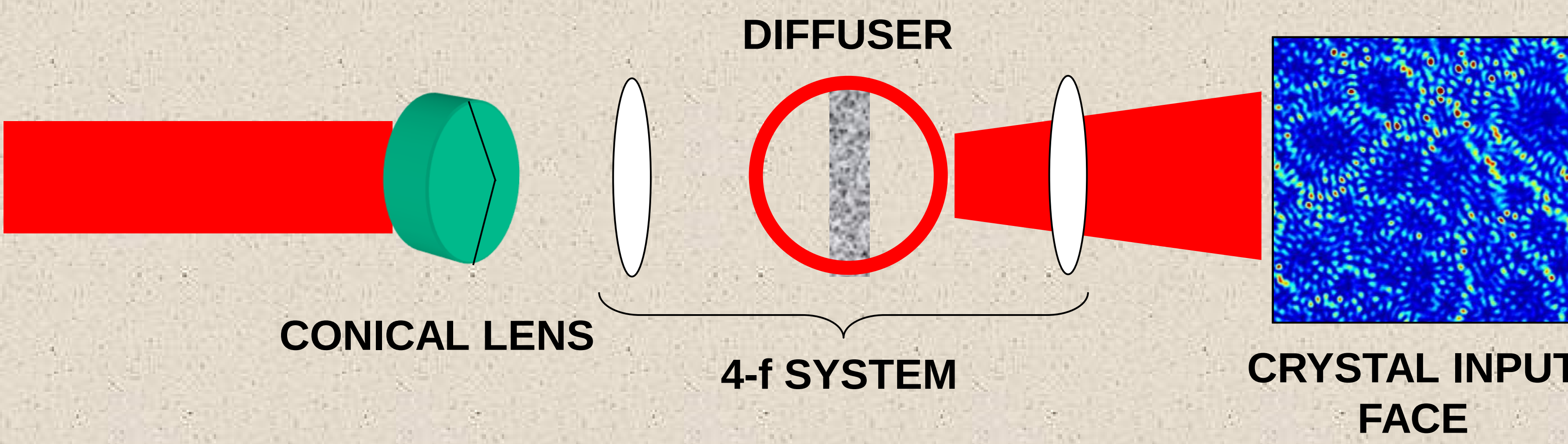
The lattice (including disorder) MUST be stationary along propagation direction

Lattice Beam
Far-Field



$$k_z^{\;2} = k - \left(k_x^{\;2} + k_y^{\;2}\right)$$

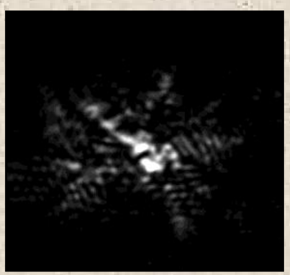
All Fourier components travel with the same velocity



Rotating the diffuser – different realization, same statistics

Experimental Results

statistical problem: must take *ensemble average* (many realizations of disorder)



Measure of Confinement:

(for output intensity distribution)

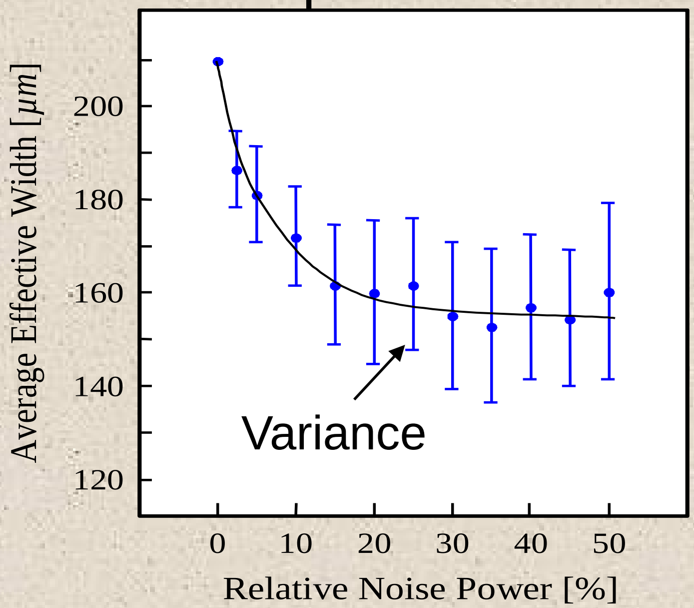
$$\gamma = \frac{\int |\Psi|^4 ds}{\left(\int |\Psi|^2 ds\right)^2} \left[1/area\right]$$

aka: inverse participation ratio

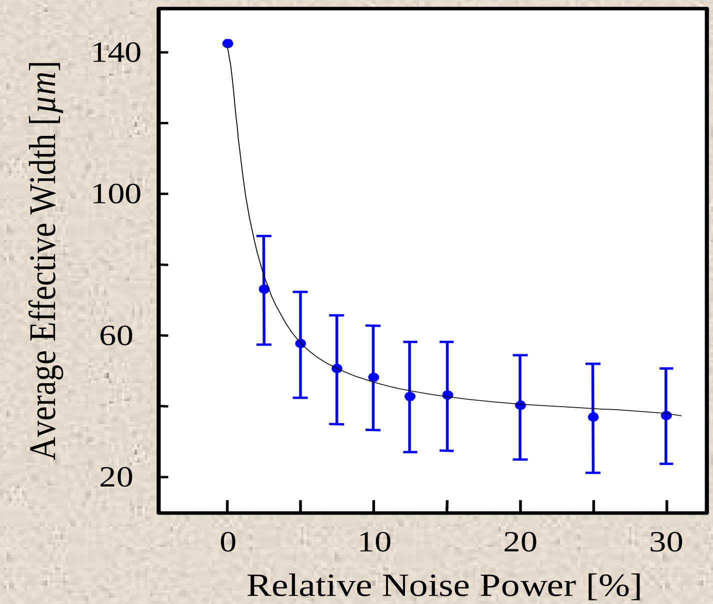
Averaged Effective Width: $$\omega_{eff} = \left\langle \frac{1}{\sqrt{\gamma}} \right\rangle$$

Averaged over 100 realizations of disorder

Experiments



Numerics

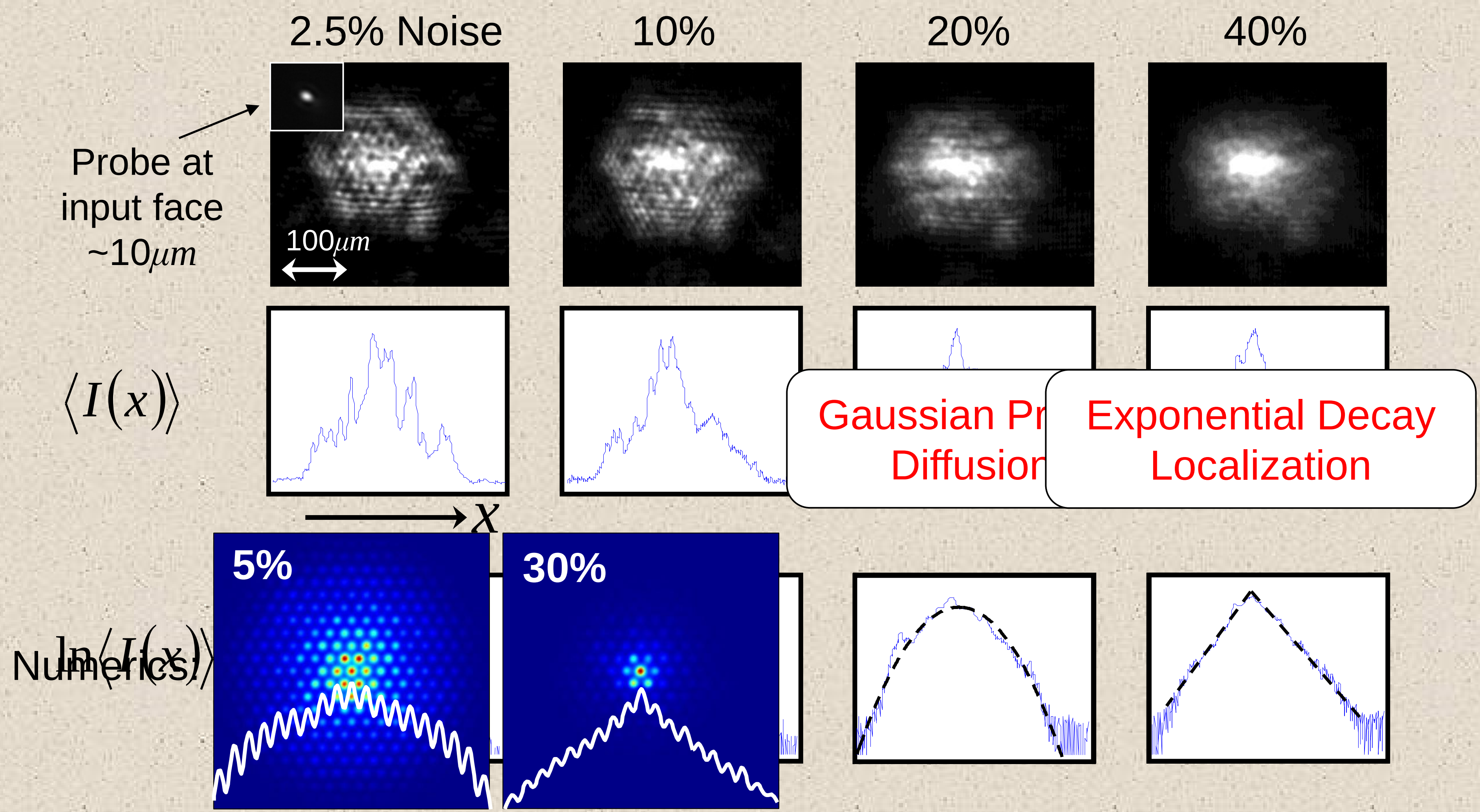


First observation of Anderson Localization in any periodic system

Experimental Observation of Anderson Localization

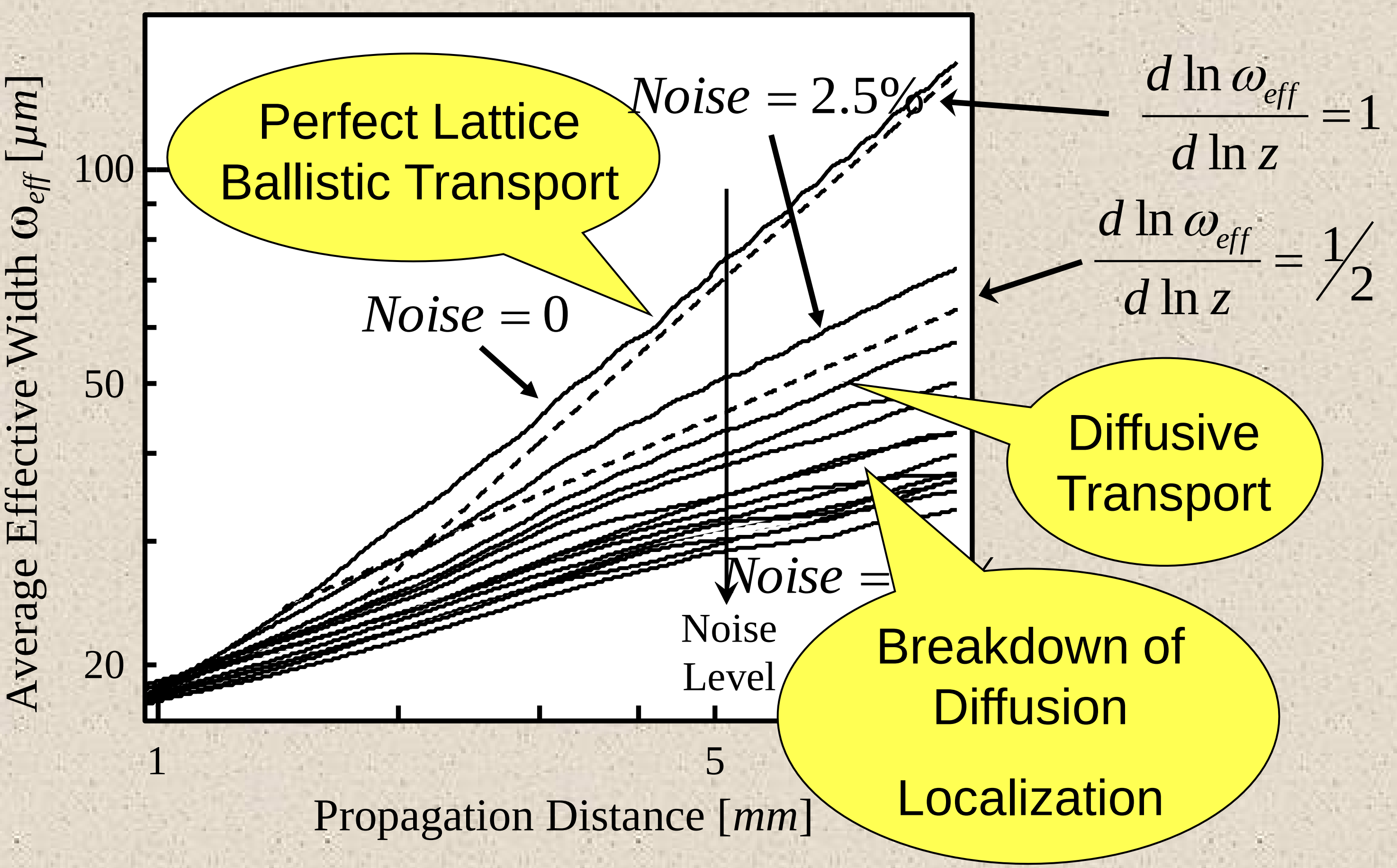
It's a statistical problem: must take *ensemble average* (many realizations of disorder)

Averaged output intensity cross-section (100 samples) at crystal output face (10*mm* propagation)



Transport Dynamics Along Propagation

We follow the (ensemble-averaged) diffraction of the beam inside the disordered lattice:



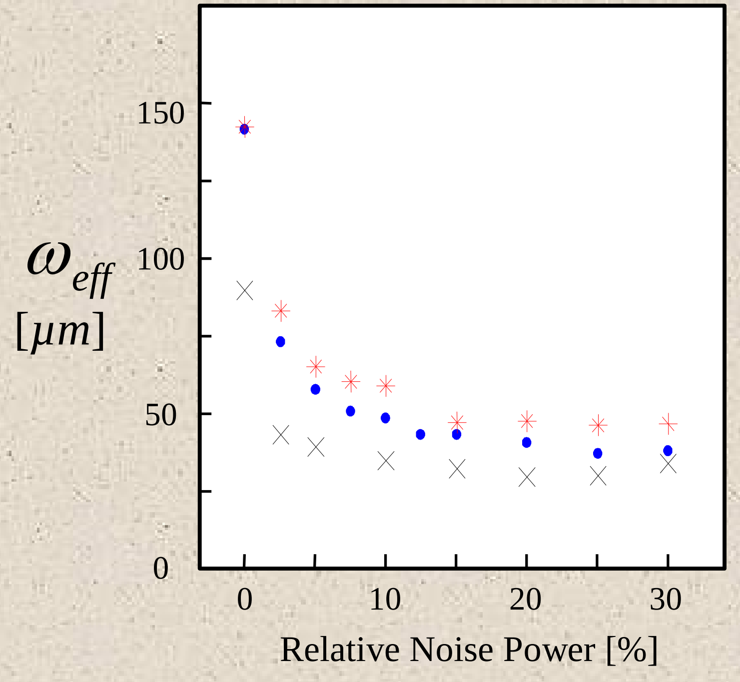
First observation of Anderson Localization in any periodic system

Disorder and Nonlinearity

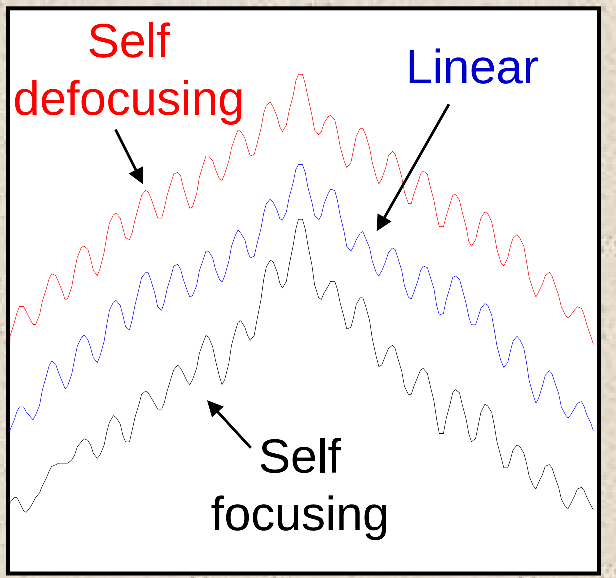
Nonlinearities (Coulomb interactions, nonlinear optical response) may play a fundamental role in disordered systems

Thus far, the combined effect (NL + disorder) is yet unclear

In our system: (numerics)

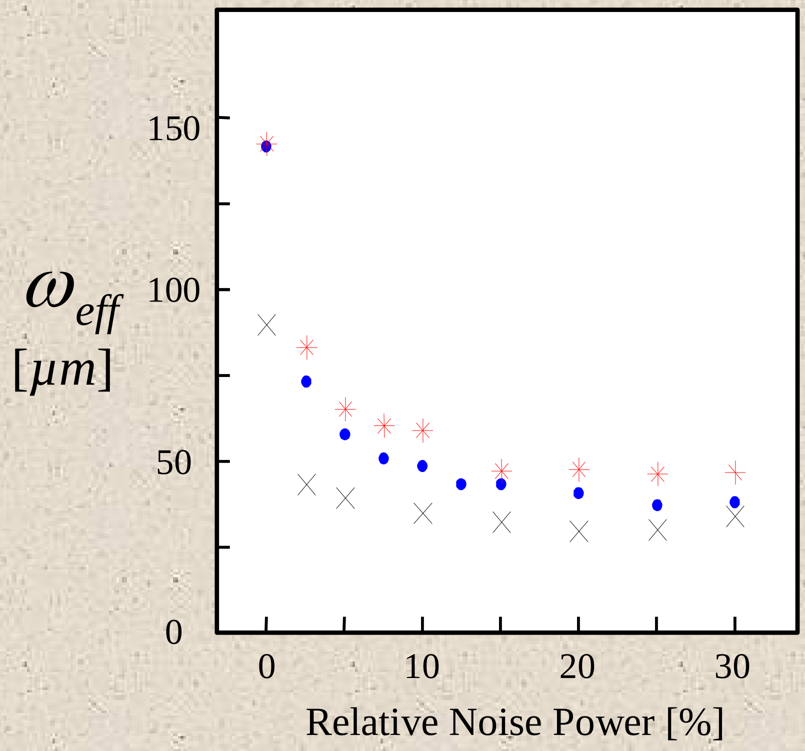


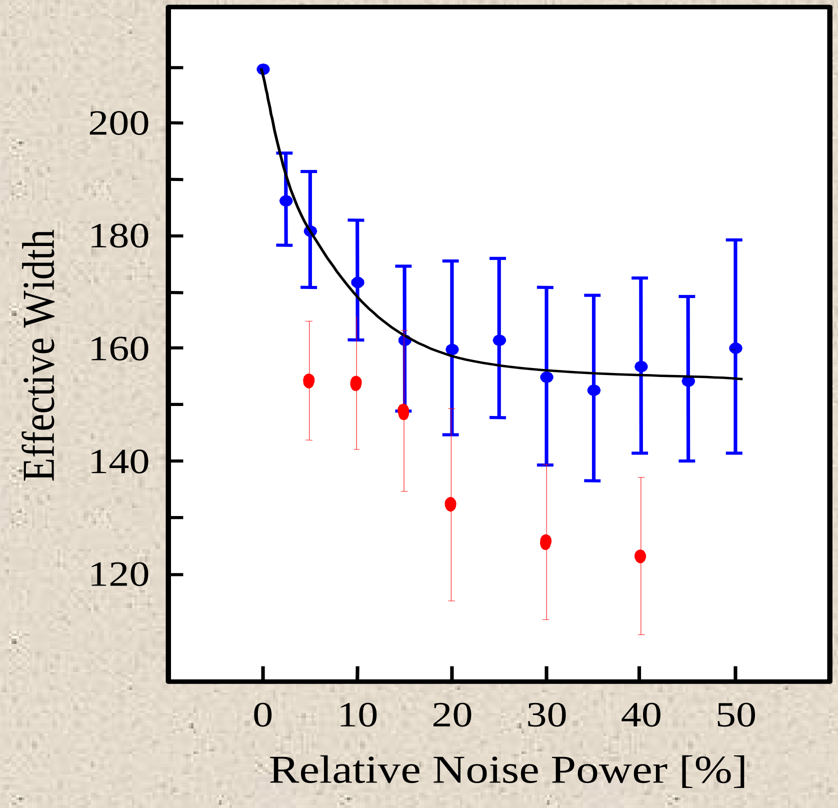
$\ln\langle I(x)\rangle$ @ 15% Noise



Experiments are in process…

Disorder and Nonlinearity: Experiments!

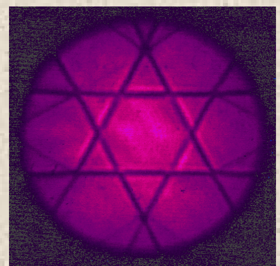


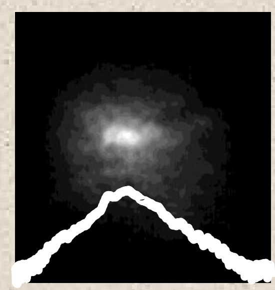


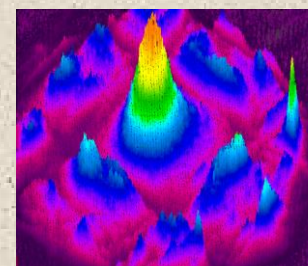
First experiments ever on
Anderson Localization + nonlinearity

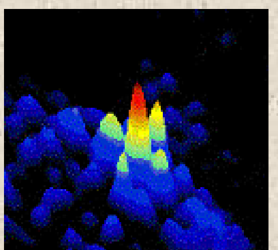
Summary

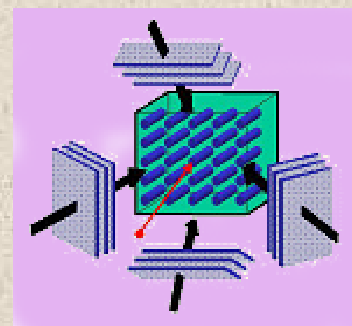
Studying universal physics of nonlinear periodic systems

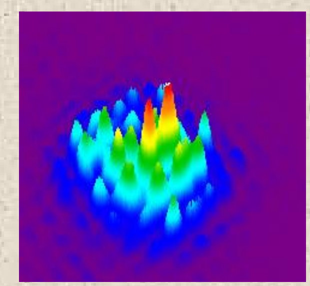


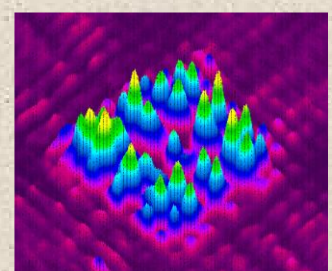


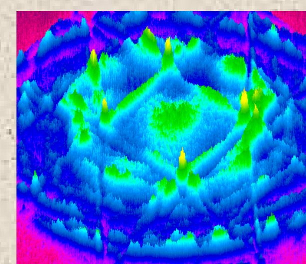


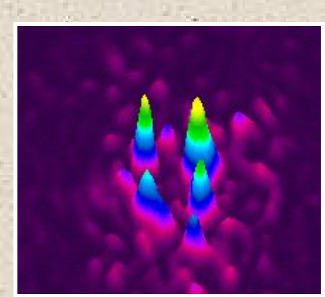








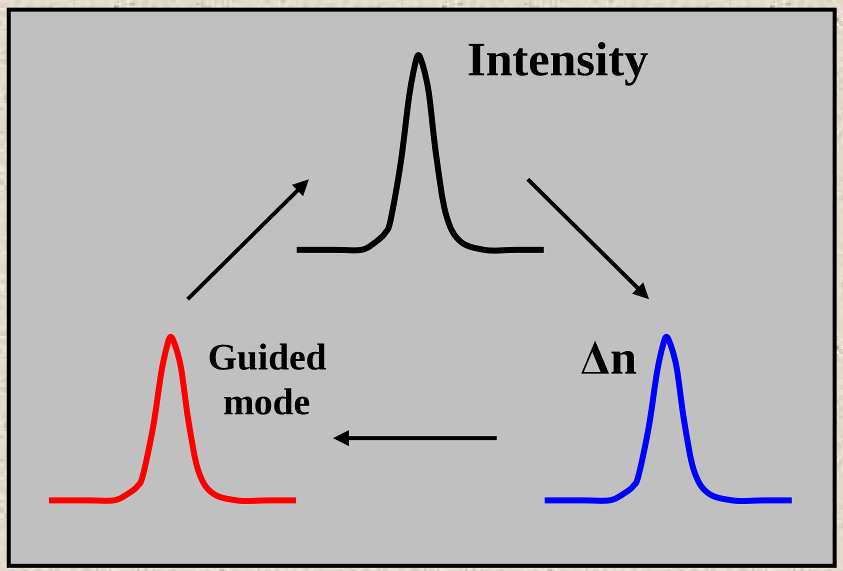




Finding Lattice Solitons: the self-consistency method

Homogeneous

The soliton is a guided mode of its own induced waveguide



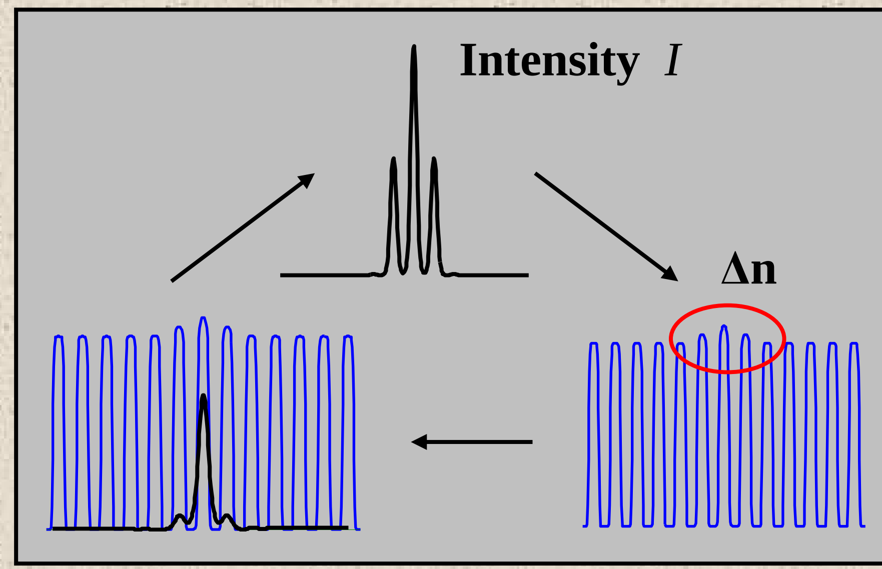
<u>Idea</u>: Askar'yan, 1962

<u>Formulation</u>: Snyder *et al*. 1991

<u>First use</u>: Mitchell, Segev, Christodoulides 1997

Lattice

The soliton is a localized mode of the full potential (lattice + induced defect)



Localized mode

$$\left[\frac{\partial^2}{\partial x^2} + V(x) + I\right]\Phi = \beta\Phi$$

Full potential (lattice + induced defect)

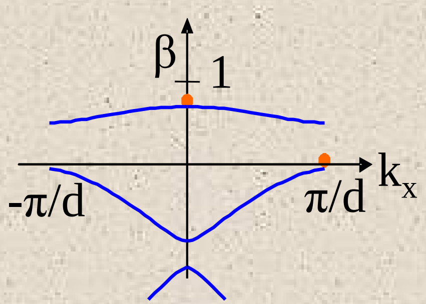
$$I = |\Phi|^2$$

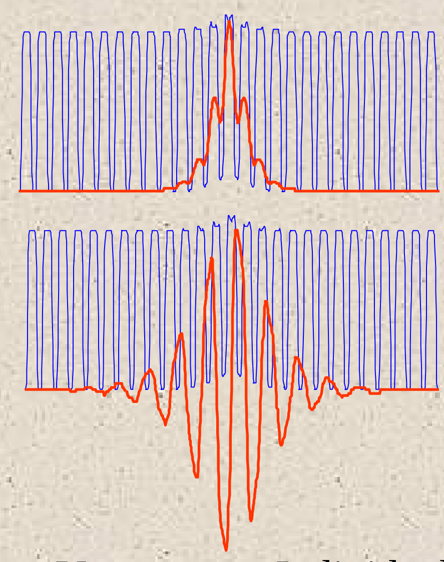
$$\Delta n = \Delta n(I)$$

O. Cohen *et al*. PRL **91**, 113901 (2003).

Multi-band vector lattice solitons

Self-focusing nonlinearity

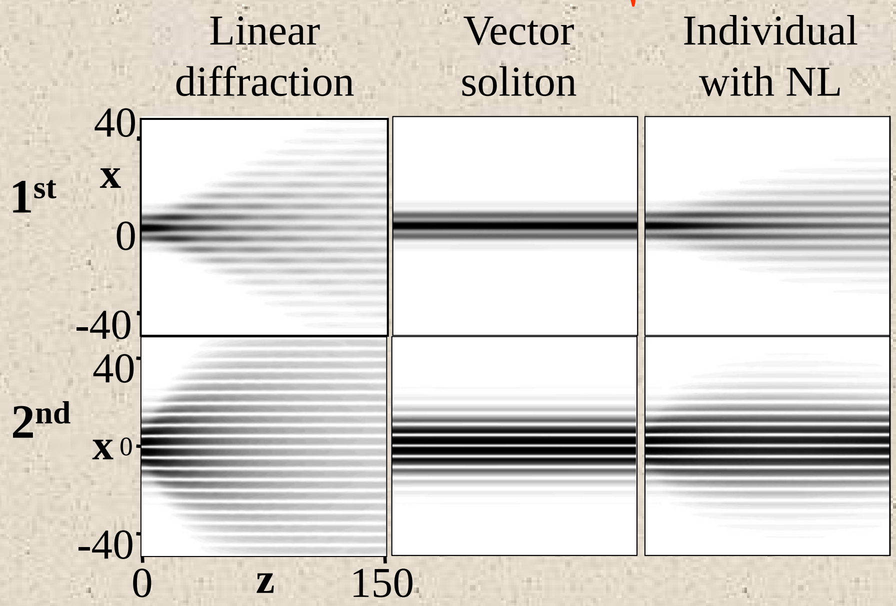




Richer dynamics than traditional solid-state (with transitions between passive bands)

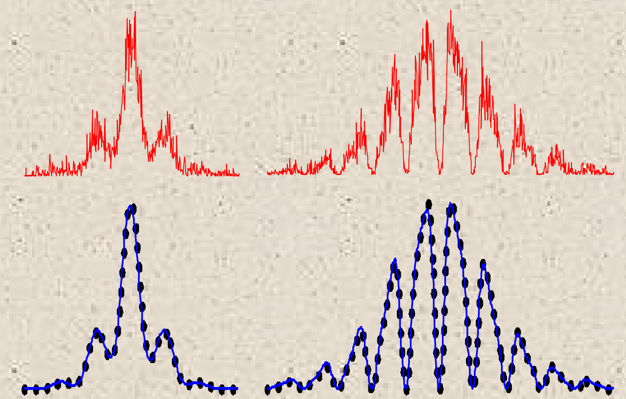
- **Both bands contribute actively to NL**
- **Bands interact with each other (through *I*)**

O. Cohen *et al*. PRL **91**, 113901 (2003)
A. Sukhorukov *et al*. PRL **91**, 113902 (2003)



BPM Results

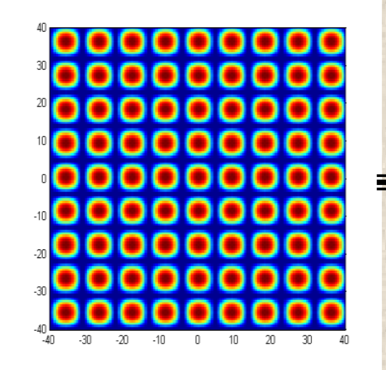
Noisy intensities at input



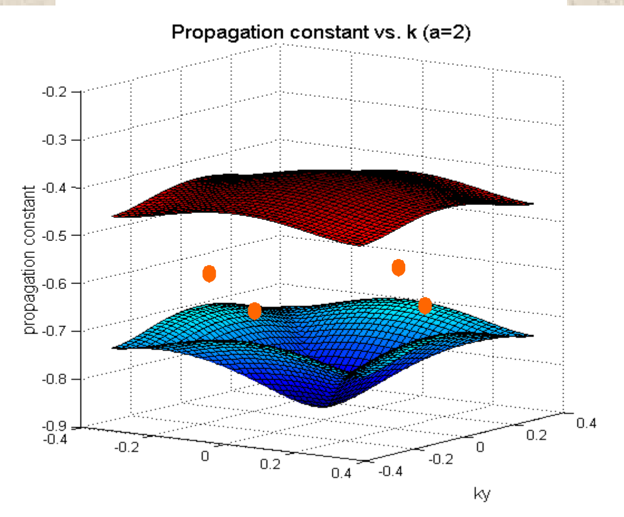
Intensities at output (on unperturbed sols.)

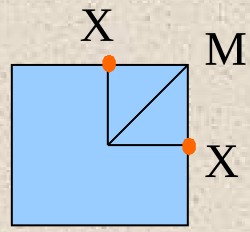
2^{nd} Band vortex lattice soliton

2D Lattice



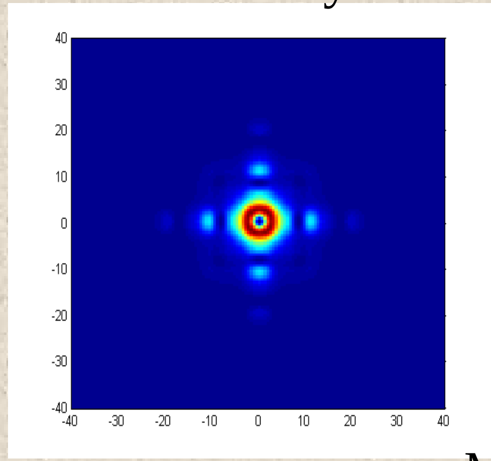
Transmission spectrum



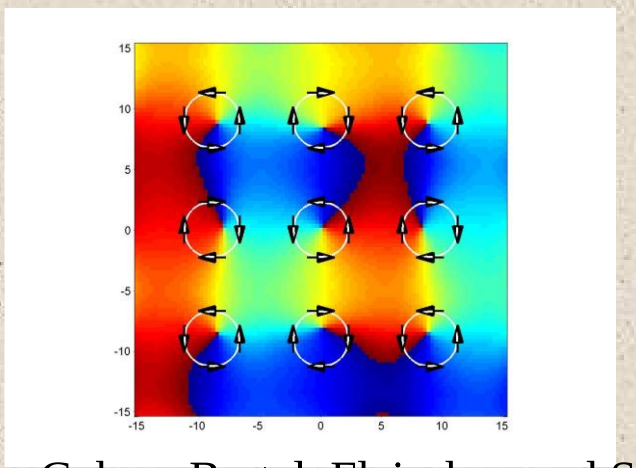


In the 2^{nd} band the edge of BZ is at the X points

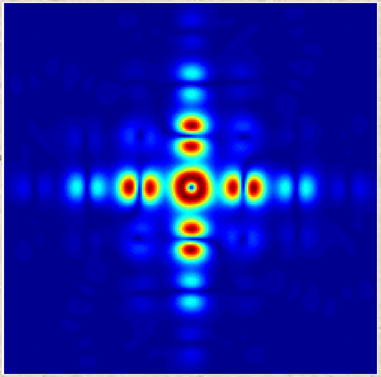
Soliton intensity



Phase structure – array of counter-rotating vortices



Diffraction intensity

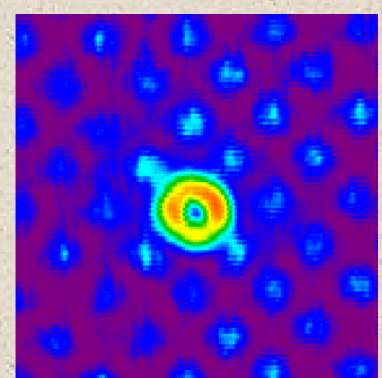


Manela, Cohen, Bartal, Fleischer, and Segev.
Opt. Lett. 17, 2049 (2004)

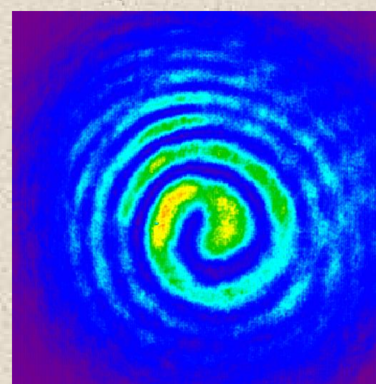
Evolution from a ring to a 2nd Band vortex lattice soliton

Excitation (input)

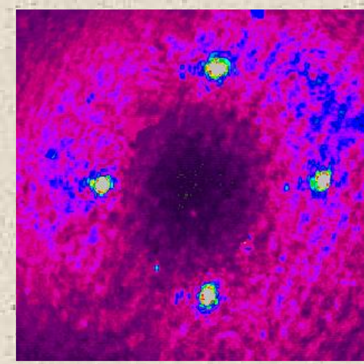
Input intensity



Input phase (interferogram)

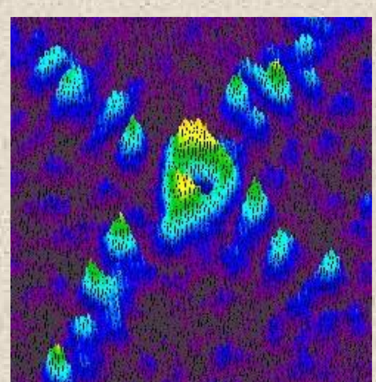


Power spectrum

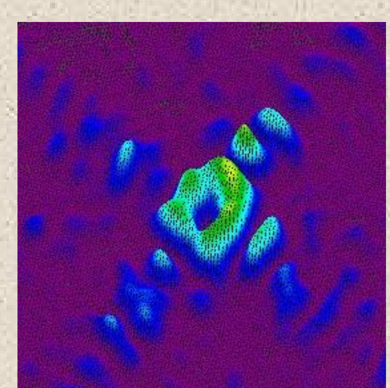


Output

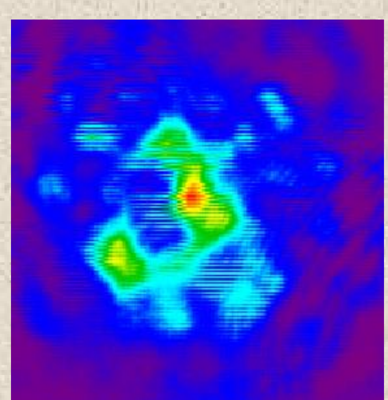
Diffraction



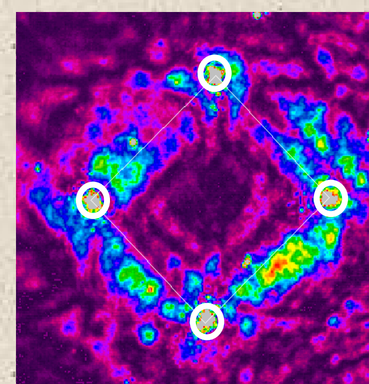
Soliton intensity



Soliton phase (interferogram)



Power spectrum

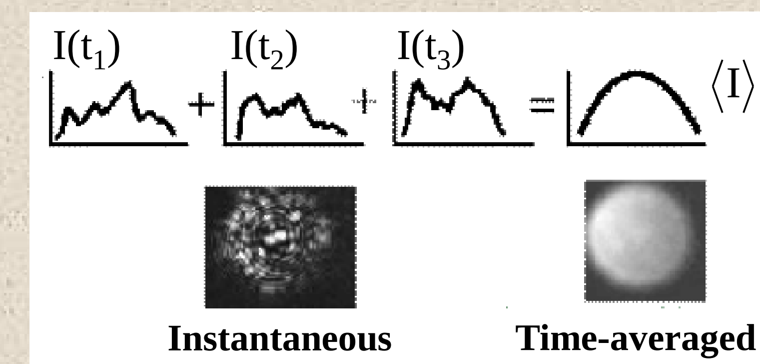


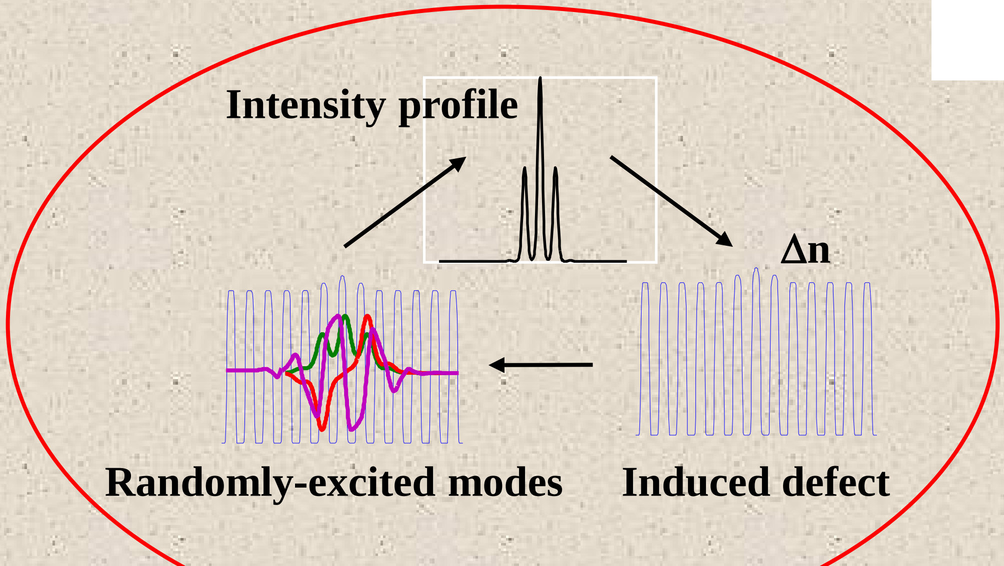
Requirements for random-phase lattice solitons

Soliton requires Δn(x,z,t) = Δn(x), *i.e. Δn independent of z and t.*

Incoherent light: $I = \tilde{I}(t)$

- Instantaneous NL => Δn(t)
- Non-instantaneous NL => Δn(<I>)





Self-consistency loop

▯ **Need to use** Δn(<I>)

Know NL modes from Multiband Vector Lattice Solitons

PRL **91**, 113901 (2003).

Incoherent light excites statistical ensemble of modes

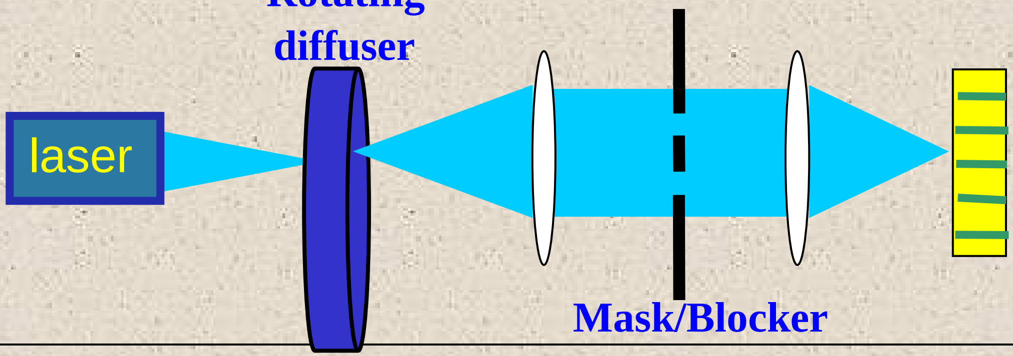
$$\psi_i(x,t) = \tilde{c}_i(t)\Phi_i(x)$$

$$\langle I(t)\rangle = \sum |\tilde{c}(t)_i|^2 \Phi_i^2(x)$$

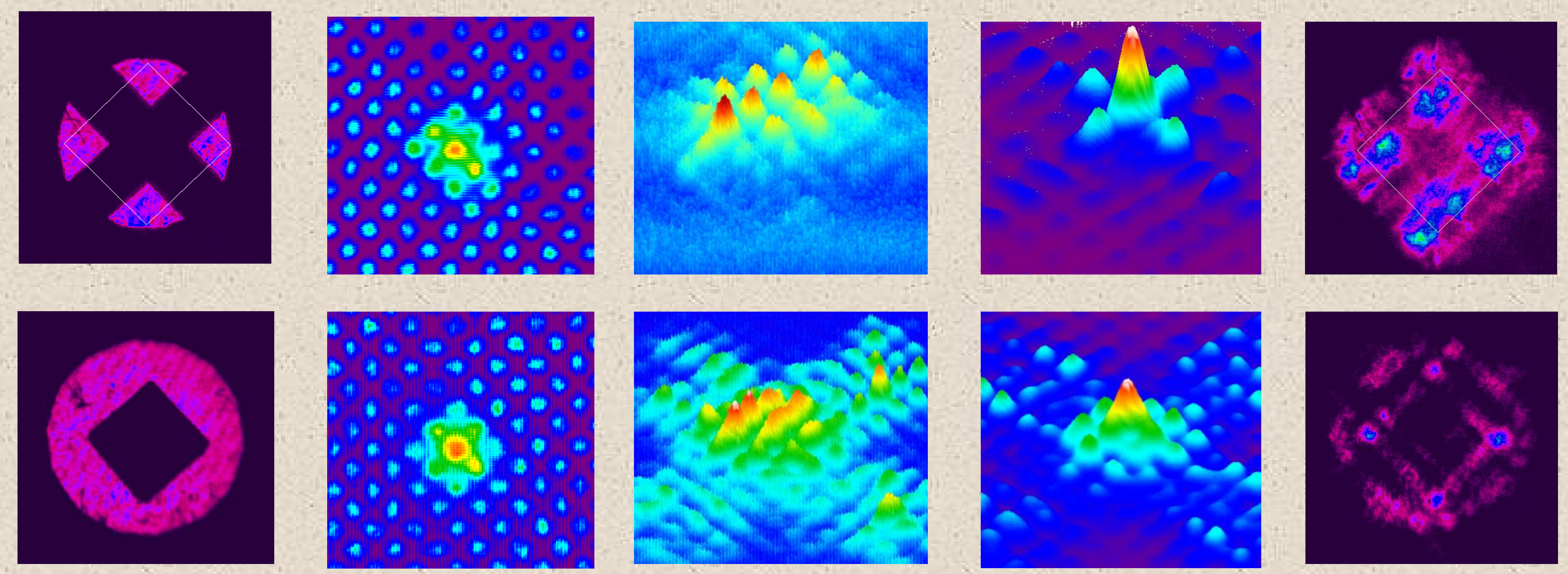
Random-Phase Gap Lattice Solitons:

RPLS arising from anomalous diffraction regions + self-defocusing nonlinearity

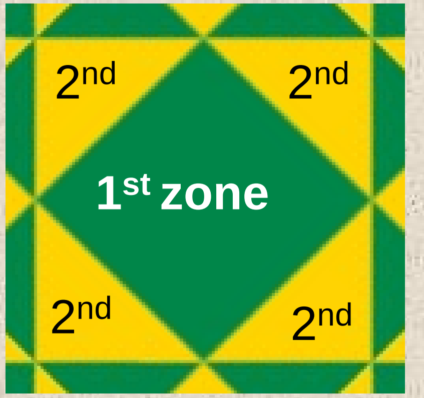
Bartal *et al.* , Opt. Lett. 2006

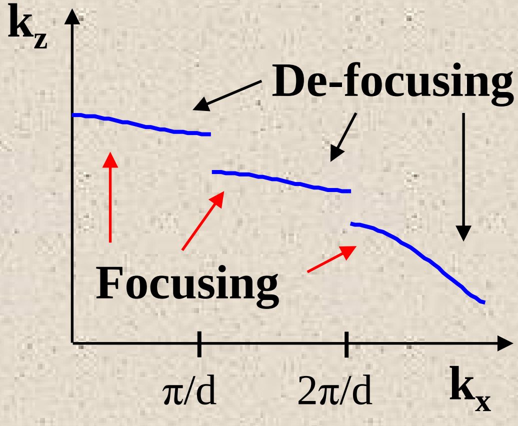


Input Fourier space	Input real space	Lattice Diffraction	Soliton Real space	Soliton Fourier space



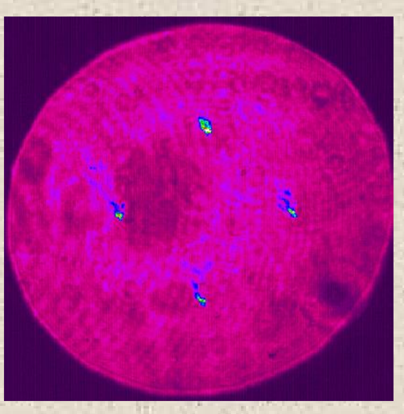
Bloch-wave spectroscopy: Nonlinear effect



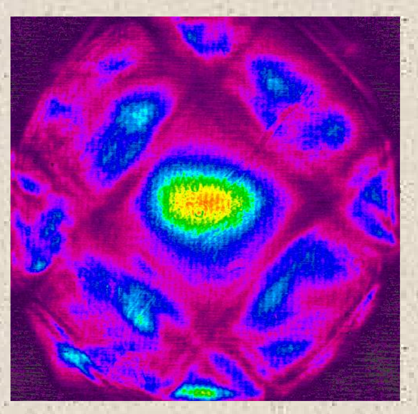


Nonlinearity:

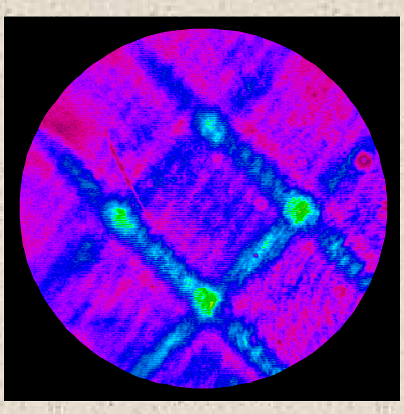
- Exchanges energy between linear modes
- Sensitive to band curvature
- Maps regions of different transport



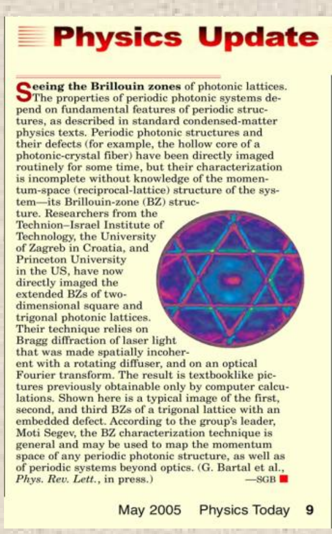
Nonlinearity off



Self focusing nonlinearity



De-focusing nonlinearity



Physics Update

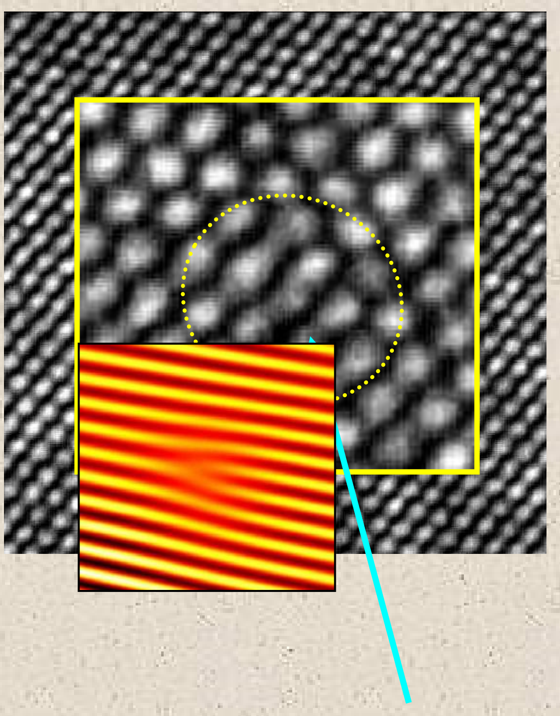
Seeing the Brillouin zones of photonic lattices. The properties of periodic photonic systems depend on fundamental features of periodic structures, as described in standard condensed-matter physics texts. Periodic photonic structures and their defects (for example, the hollow core of a photonic-crystal fiber) have been directly imaged routinely for some time, but their characterization is incomplete without knowledge of the momentum-space (reciprocal-lattice) structure of the system—its Brillouin-zone (BZ) structure. Researchers from the Technion–Israel Institute of Technology, the University of Zagreb in Croatia, and Princeton University in the US, have now directly imaged the extended BZs of two-dimensional square and trigonal photonic lattices. Their technique relies on Bragg diffraction of laser light that was made spatially incoherent with a rotating diffuser, and on an optical Fourier transform. The result is textbooklike pictures previously obtainable only by computer calculations. Shown here is a typical image of the first, second, and third BZs of a trigonal lattice with an embedded defect. According to the group's leader, Moti Segev, the BZ characterization technique is general and may be used to map the momentum space of any periodic photonic structure, as well as of periodic systems beyond optics. (G. Bartal et al., *Phys. Rev. Lett.*, in press.) —SGB

May 2005 Physics Today 9

G. Bartal *et al*. PRL 94, 163902 (2005).

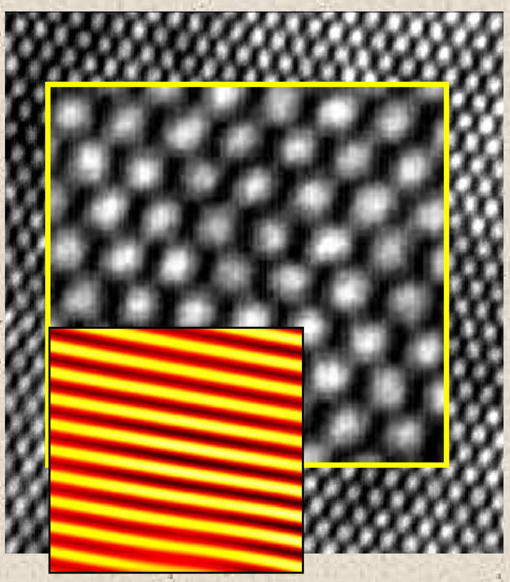
Experiments with hexagonal lattice defects

Non-interacting lattice, with dislocation



Dislocation

Interacting lattice – dislocation removed

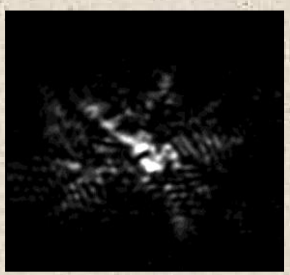


"Pure" lattice; Dislocation removed

Output face pictures

Experimental Results

statistical problem: must take *ensemble average* (many realizations of disorder)



Measure of Confinement:

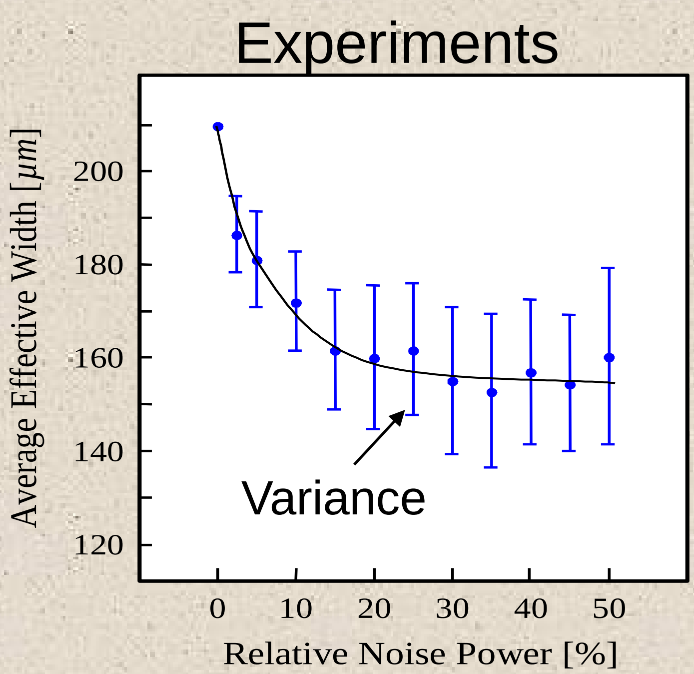
(for output intensity distribution)

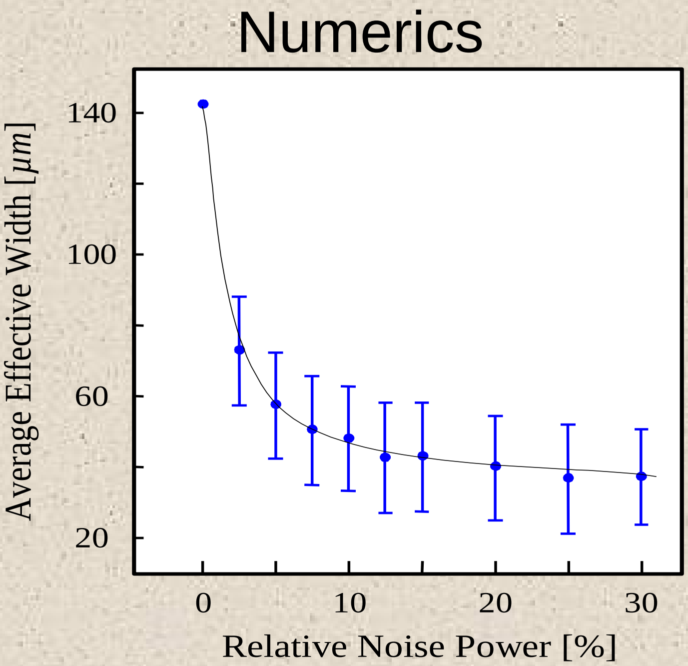
$$\gamma = \frac{\int |\Psi|^4 ds}{\left(\int |\Psi|^2 ds\right)^2} \left[1/area\right]$$

(*aka: inverse participation ratio*)

Averaged Effective Width: $$\omega_{eff} = \left\langle \frac{1}{\sqrt{\gamma}} \right\rangle$$

Averaged over 100 realizations of disorder





First observation of Anderson Localization in any periodic system